

## Week 4 – Statistics, Part 2 (5/11/20 –5/15/20)

Welcome to distance learning. Assignments are required from now until then end of the school year. You will be graded on submitted material.

**Goal:** To understand Different ways to summarize data both numerically and graphically. To compare two data sets by using these statistical measures.

Office Hour Schedule	
Monday	Notes: Statistical Measures
Tuesday	Lesson 15.1
Wednesday	Lesson 15.2
Thursday	Lesson 15.3
Friday	Open Question and Answer

Contact		
Office hours by Email:	Mon – Fri: 8:00 AM – 3:30 PM mdibley@tusd.net	
Office hours by video:	Mon – Fri: 10:00 – 10:30 AM <a href="https://zoom.us/j/312003066">https://zoom.us/j/312003066</a>  Meeting ID: 312 003 066 Password: 805373	Mon – Fri: 3:00 – 3:30 PM <a href="https://zoom.us/j/218432703">https://zoom.us/j/218432703</a>  Meeting ID: 218 432 703 Password: 672048

How to get/return an assignment:	
Digital Option	non-Digital Option
<ul style="list-style-type: none"> <li>All digits lessons can be accessed through your digits account.</li> <li>Videos, Notes, Content Practice (homework), etc. will all be uploaded to digits on (or before) Monday, May 11.</li> <li>All practice problems from the digits lessons may be completed on paper or in a notebook.</li> <li>Digital assignments are submitted in the normal way.</li> <li>Worksheets may be photographed and emailed or uploaded to digits.</li> </ul>	<ul style="list-style-type: none"> <li>Lessons will be provided in a paper format.</li> <li>A packet must be picked up from the George Kelly office on Friday May 8. This is the last pick-up day current scheduled.</li> <li>Completed assignments must be returned to the George Kelly office on Friday, May 15. This is the last drop-off day current scheduled.</li> </ul>

### Digital Option

1. (*digits* Topic 15) Comparing Two Populations
  - a. Video: “Mean, Median and Mode” (<https://youtu.be/B1HEzNTGeZ4>)
  - b. Video: “Box and Whisker Plots” (<https://youtu.be/8nVFChsPG3g>)
  - c. Notes: “Statistical Measures”
  - d. Lesson 15-1: “Statistical Measures” (view the lesson and answer the Got It? Problems)
  - e. Lesson 15-2: “Multiple Populations and Inferences” (view the lesson and answer the Got It? Problems)
  - f. Lesson 15-3: “Using Measures of Center” (view the lesson and answer the Got It? Problems)
2. Content Practice
  - a. Statistical Measures WS
  - b. 15-1 Homework K
  - c. 15-2 Homework K
  - d. 15-3 Homework K
3. Bonus Logic Problem: Checkerboard
  - a. This one is quite challenging. I suggest you try a smaller problem, like a 3x3 checkerboard first. (or, maybe a 2x2. Or ... how about a 1x1?)
  - b. If you would like a clue, just ask.

Unit 5

5/11/20

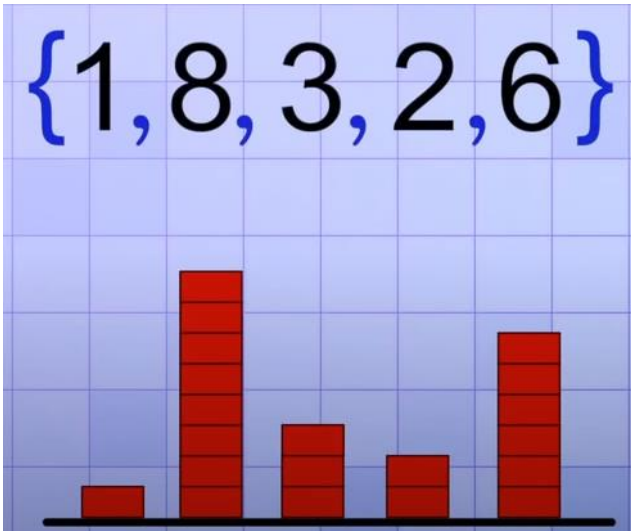
## Statistical Measures

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- Write the date and unit in the top corners of the page
- Write the title across the top line

# Mean (or average)

The center of a numerical data set.

- To find the mean: sum the data values and then divide by the number of values in the data set.



The mean tells you what value you would get if every all of the numbers “borrowed” from each other, until they were all the same.

Restack the blocks until all stacks are the same height.



# Median

Remember this, it is  
SUPER important.



The center of a numerical data set.

- To find the median: arrange the data values in numerical order.
  - For an odd number of data values, the median is the middle value.
  - For an even number of data values, the median is the average of the two middle values.

# Mode

The value that appears most often in a data set.

# Range

A measure of variability of a numerical data set. (The “spread” of a numerical data set).

- To find the range: find the difference between the maximum (greatest) and minimum (least) values.

## Example 1

Data set: 7, 9, 8, 9, 16, 7, 8, 7, 10

$$\text{Mean: } \frac{7 + 9 + 8 + 9 + 16 + 7 + 8 + 7 + 10}{9} = \frac{81}{9} = 9$$

$$\text{Median: } 7, 7, 7, 8, 8, 9, 9, 10, 16 = 8$$

$$\text{Mode: } 7, 7, 7, 8, 8, 9, 9, 10, 16 = 7$$

$$\text{Range: } 16 - 7 = 9$$

## Example 2

Data set: 5, 8, 5, 12, 7, 11

$$\text{Mean: } \frac{5 + 8 + 5 + 12 + 7 + 11}{6} = \frac{48}{6} = 8$$

$$\text{Median: } 5, 5, \boxed{7, 8}, 11, 12 = \frac{7 + 8}{2} = \frac{15}{2} = 7.5$$

$$\text{Mode: } \boxed{5, 5}, 7, 8, 11, 12 = 5$$

$$\text{Range: } 12 - 5 = 7$$

Unit 5

5/11/20

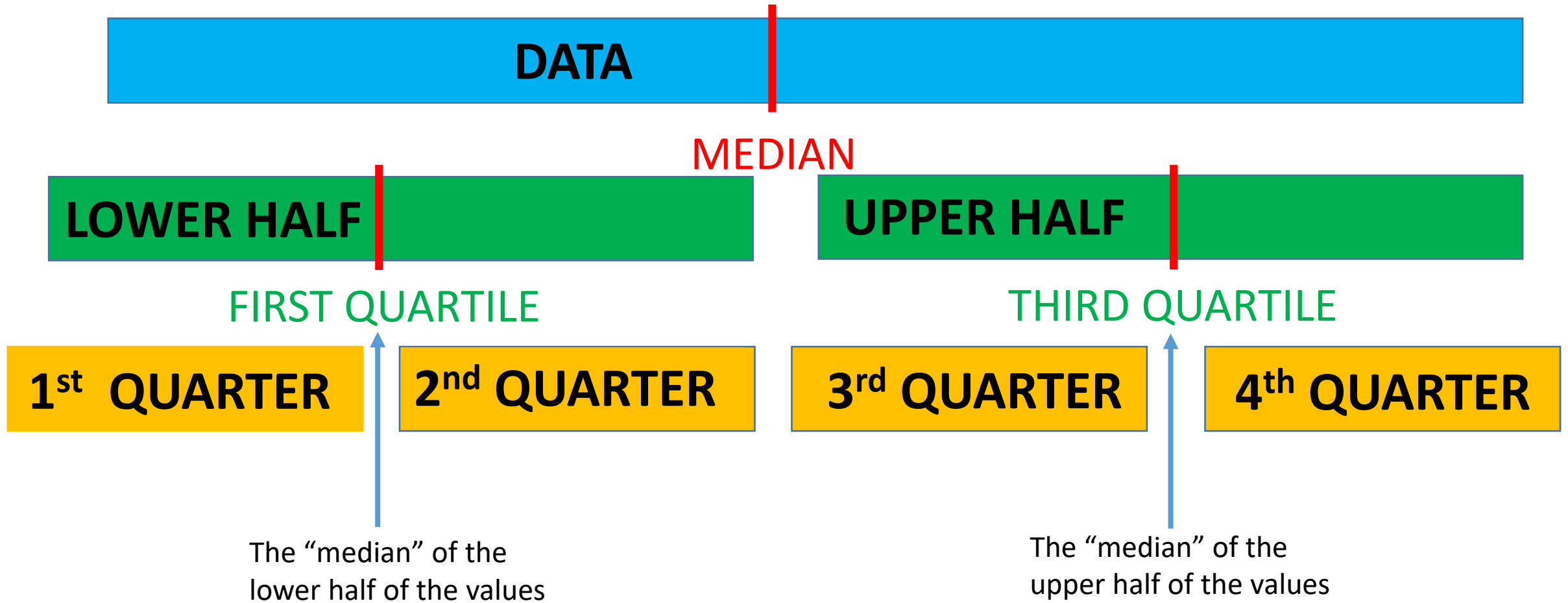
## Statistical Measures, Part II

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- Write the title across the top line



# Quartile

Quartiles divide the data set into four parts, each the same size.



# Quartile

A measure of variability of a numerical data set.

- Find the median of the data set.
- First quartile: find the “median” of the values less than the median.
- Third quartile: find the “median” of the values greater than the median.

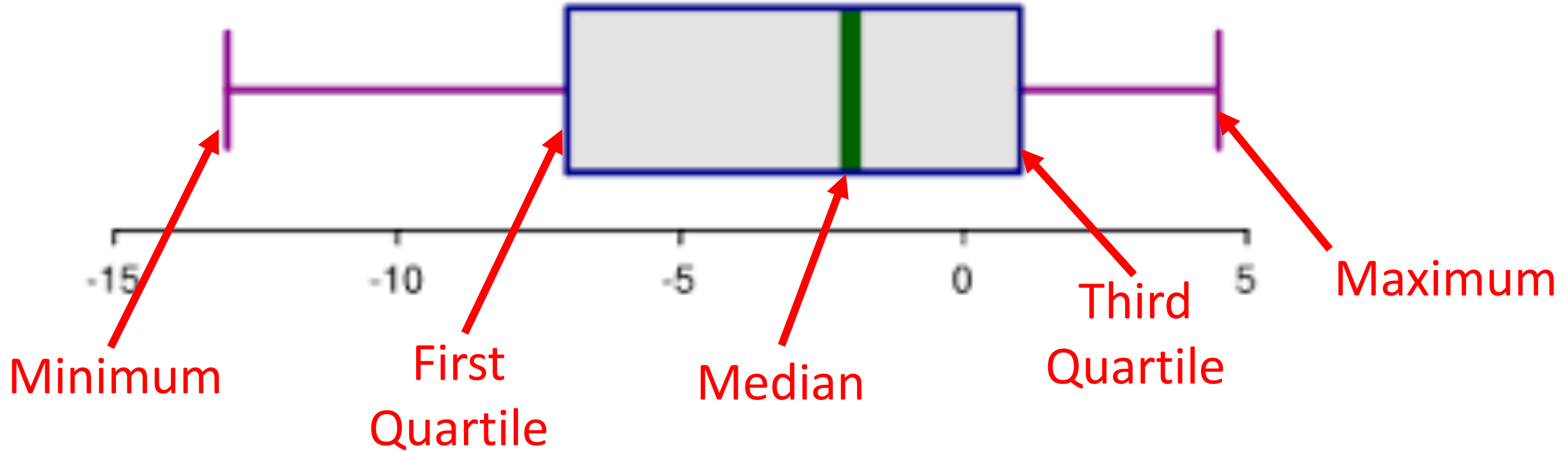
# Interquartile Range (IQR)

A measure of variability of a numerical data set.

- Find the difference between the third quartile and the first quartile.

# Boxplot

A graph that shows the distribution of a data set by marking five boundary points, as shown below.



# Example 1 - Make a Boxplot

Data set: 7, 9, 8, 9, 16, 7, 8, 7, 10, 13, 5

Minimum: 5, 7, 7, 7, 8, 8, 9, 9, 10, 13, 16 = 8 Maximum

First Quartile: 5, 7, 7, 7, 8 = 7

Third Quartile: 9, 9, 10, 13, 16 = 10

Interquartile Range:  $10 - 7 = 3$

To make a Boxplot you need to find five values:

1. Minimum (least value)
2. Maximum (greatest value)
3. Median
4. 1<sup>st</sup> Quartile
5. 3<sup>rd</sup> Quartile

## Example 1 - Make a Boxplot (continued)

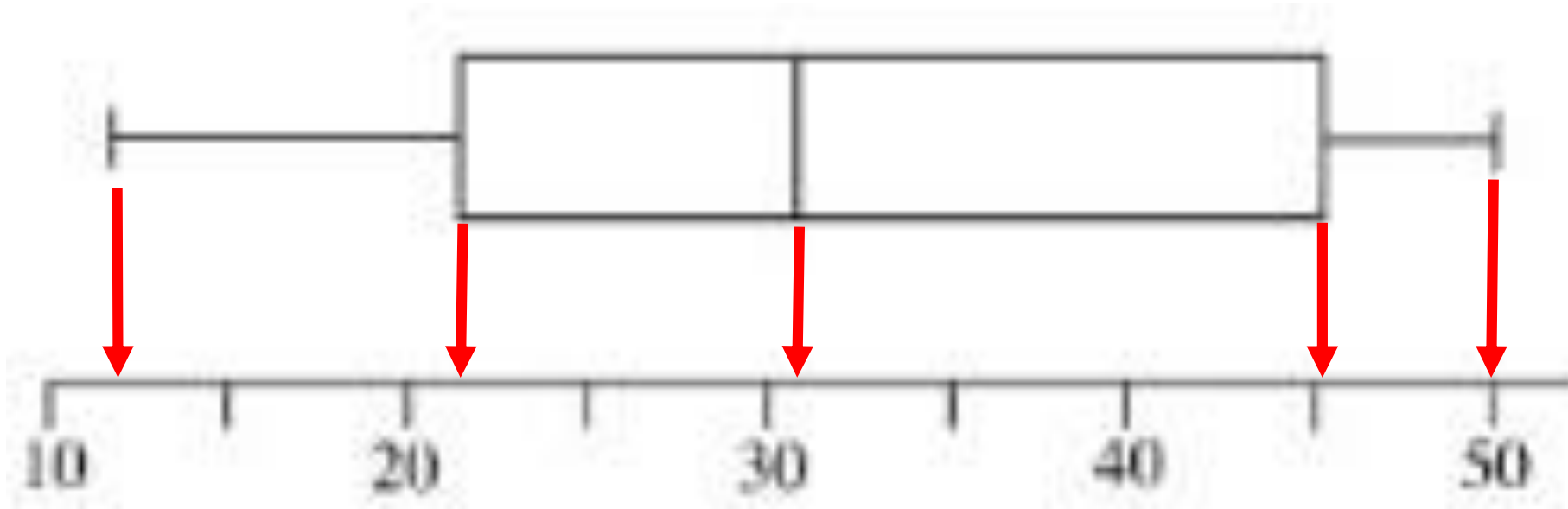
Plot the five boundary conditions on the number line and then draw a box using the middle three.

We found these numbers on the previous slide:

min=5, first quartile=7, median=8, third quartile=10, max=16



## Example 2 – Reading a Box Plot



Minimum: 12

First Quartile: 22

Median: 31

Third Quartile: 45

Maximum: 50

Range:  $50 - 12 = 38$

IQR:  $45 - 22 = 23$

# 15-1: Statistical Measures

## Part 1

### Intro

A measure of center is a single, central value that summarizes a set of data.

The **mean** of a set of data values is the sum of the data divided by the number of data values.

data set: -5, -1, 3, 6, 8, 8, 9

$$\begin{aligned}\text{mean} &= \frac{\text{sum of the data values}}{\text{the number of data values}} \\ &= \frac{-5 + -1 + 3 + 6 + 8 + 8 + 9}{7} \\ &= \frac{28}{7} \\ &= 4\end{aligned}$$

The mean is a measure of center.

For an odd number of data values, the **median** is the middle value when the data values are arranged in numerical order.

data set: -5, -1, 3, 6, 8, 8, 9

↑  
median

The median is a measure of center.

For an even number of data values, the **median** is the average of the two middle items when the data values are arranged in numerical order.

data set: -5, -1, 3, 6, 7, 8, 8, 9

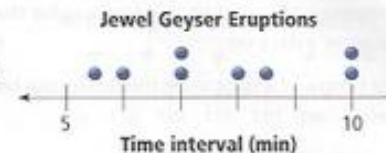
$$\begin{aligned}\text{median} &= \frac{6 + 7}{2} \\ &= \frac{13}{2} \\ &= 6.5\end{aligned}$$

The median is a measure of center.

## Part 1

### Example Finding Mean and Median Intervals

At Yellowstone National Park, Data Girl and Ms. Adventure watch Jewel Geyser erupt. Data Girl records the time intervals between eruptions.



- Find the mean and median intervals between eruptions.
- Using the measure of center, what inference can you make about how often Jewel Geyser erupts?

### Solution

- Find the mean interval.

There are eight values in the data set. Use the formula for the mean.

$$\begin{aligned}\text{mean} &= \frac{\text{sum of the data values}}{\text{the number of data values}} \\ &= \frac{5.5 + 6 + 7 + 7 + 8 + 8.5 + 10 + 10}{8} \\ &= \frac{62}{8} \\ &= 7.75\end{aligned}$$

$0.75 = \frac{3}{4}$ , and  $\frac{3}{4}$  of a minute is 45 seconds.

The mean interval between eruptions is 7 minutes 45 seconds.

Find the median interval.

Make sure the time intervals are listed in order.

5.5, 6, 7, 7, 8, 8.5, 10, 10

There is an even number of data values. Take the average of the middle two values to find the median.

$$\begin{aligned}\text{median} &= \frac{7 + 8}{2} \\ &= 7.5\end{aligned}$$

The median interval between eruptions is 7 minutes 30 seconds.

- Sample: The mean and median are close together. The dot plot shows that the two values at 10 minutes are higher than the rest of the data, so the median may describe the more typical central value. You can make the inference that Jewel Geyser erupts about every 7 minutes and 30 seconds.

The data set for this dot plot is:  
5.5, 6, 7, 7, 8, 8.5, 10, 10

Be sure you understand why there are two 7s and two 10s.



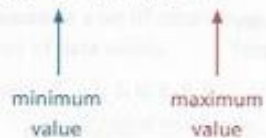
## Part 2

### Intro

A measure of variability is a single value that describes the spread of values in a data set.

The **range** of a data set is the difference between the greatest and the least values.

data set: -5, -1, 4, 7, 8, 8, 11



$$\text{range} = \text{maximum value} - \text{minimum value}$$

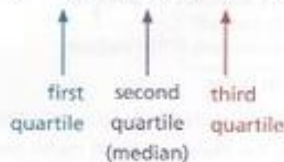
$$= 11 - (-5)$$

$$= 11 + 5$$

$$= 16$$

The **quartiles** of a data set divide the data set into four parts with the same number of data values in each part.

data set: -5, -1, 4, 7, 8, 8, 11



The **interquartile range (IQR)** is the difference between the first and third quartiles of the data set. It represents the spread of the middle 50% of the data values.

data set: -5, -1, 4, 7, 8, 8, 11



$$\text{interquartile range} = \text{third quartile} - \text{first quartile}$$

$$= 8 - (-1)$$

$$= 8 + 1$$

$$= 9$$

Variability:

Are the all of data points close together or are they spread out?

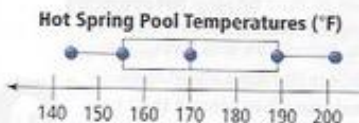
All of these values (mean, median, range, etc) are in your notes.

Also, you really should watch the two videos listed in your instructions.

## Part 2

### Example Finding Ranges and Interquartile Ranges

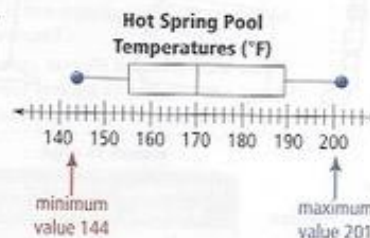
Ms. Adventure and Data Girl are visiting different hot springs in Yellowstone. Data Girl makes a box plot of the temperatures she records.



- Find the range and interquartile range of the temperatures.
- Describe the variability of temperatures in the hot spring pools at Yellowstone.

### Solution

a.

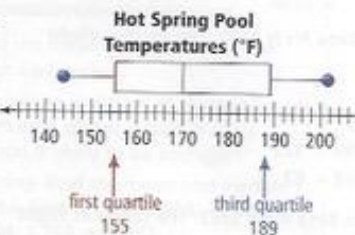


$$\text{range} = \text{maximum value} - \text{minimum value}$$

$$= 201 - 144$$

$$= 57$$

b.



$$\text{interquartile range} = \text{third quartile} - \text{first quartile}$$

$$= 189 - 155$$

$$= 34$$

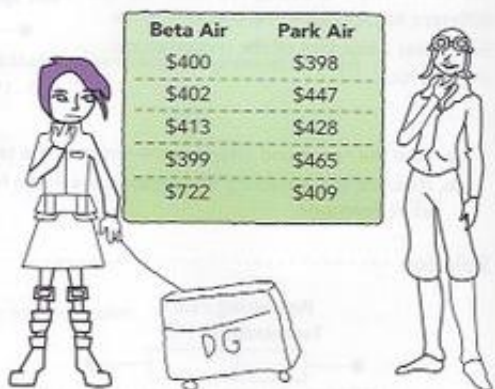
- Sample: The range is 57°F and the interquartile range is 34°F. This means that the temperatures of the hot springs are spread out evenly throughout the data set.

### Part 3

#### Example Using Median and Range to Make Inferences

Ms. Adventure and Data Girl are thinking about their next trip. They sample the flight prices of two airlines at random.

Find the median and the range for each airline. Based on the two values, make an inference about which airline they will most likely choose. Justify your reasoning.



#### Solution

**Step 1** Find the median and range.

	median
	↓
Beta Air	\$399, \$400, \$402, \$413, \$722
Park Air	\$398, \$409, \$428, \$447, \$465
	↑
	median

The median flight price on Beta Air is \$402. The median flight price on Park Air is \$428.

Range = maximum value – minimum value

Beta Air	$722 - 399 = 323$
Park Air	$465 - 398 = 67$

The range of flight prices on Beta Air is \$323. The range of flight on Park Air is \$67.

**Step 2** Make an inference.

The median is not affected by stray data values, while the range is. Since there is an unusually high price in one of the samples, use the median to make your inference.

Sample inference: The median price of Beta Air flights is lower than the median price of Park Air flights. Ms. Adventure and Data Girl will most likely choose Beta Air.

### Part 1

#### Intro

When you analyze a group, you usually have a question that you want answered about that group.

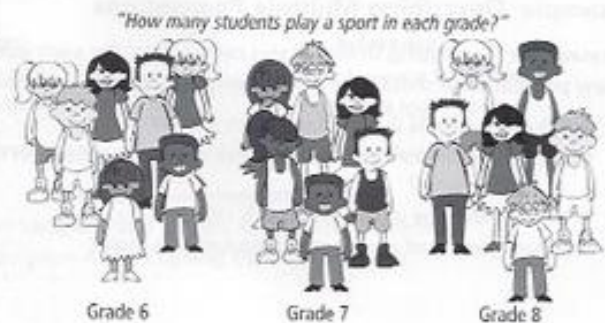
Use one population when you have a question about a trait of the whole group.



Use two populations when you want to compare two groups or two parts of a group.



Use the number of populations needed to answer the question you have.





## Part 1

### Example Determining the Number of Populations

A teacher gave a test to each of her three math classes. To answer each question, should the teacher consider the classes as *three* populations or as *one* population?

- What was the highest score overall?
- Which is the top-scoring class?
- What is the mean score in each class?
- Who is the top-scoring student in each class?
- What is the mean score of all the students?
- How does the range of scores compare among classes?

#### Solution

##### One Population

These questions ask about a *trait among all* of the students, so the classes should be considered *one combined population*.

- What was the highest score overall?
- What is the mean score of all the students?

##### Three Populations

These questions ask about *how the classes compare to each other*, so the classes should be considered *three separate populations*.

- Which is the top-scoring class?
- Who is the top-scoring student in each class?
- What is the mean score in each class?
- How does the range of scores compare between classes?

## Part 2

### Example Describing Multiple Populations

A researcher is analyzing United States census data. For each question, how many population(s) should the researcher use? Describe the population(s).

- Do more people live in Maine or in Hawaii?
- Is the median number of people per residence greater in Chicago, Los Angeles, or Philadelphia?
- What is the age of the oldest U.S. citizen?
- What percent of each state's population is male?

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## Part 2

### Example *continued*

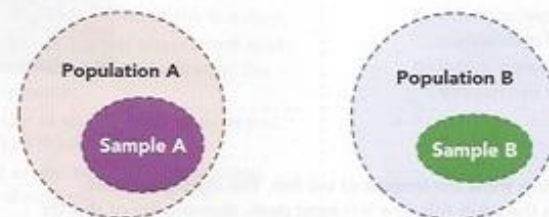
#### Solution

- The researcher should use 2 populations. Description:
  - population of Hawaii
  - population of Maine
- The researcher should use 3 populations. Description:
  - population of Chicago
  - population of Los Angeles
  - population of Philadelphia
- The researcher should use 1 population. Description:
  - the population of the entire United States
- The researcher should use 50 populations. Description:
  - the population of each of the 50 states

## Key Concept

You can make inferences about populations using random samples from each population.

When you make a judgment by interpreting a set of data, you are making an inference.



#### Inference:

Based on **Sample A**, 65% of **Population A** loves to sing.

#### Inference:

Based on **Sample B**, 30% of **Population B** loves to sing.

An inference that compares two things is called a **comparative inference**.

#### Comparative Inference:

Based on Sample A and **Sample B**, a greater percent of Population A loves to sing than Population B.

### Example Making Comparative Inferences

- Make a conjecture about why there are two peaks in the dot plots of both samples.
- Based on your conjecture, make a comparative inference about the fish in the two ponds.

Pond A		Pond B	
2	8	5	10
4	9	6	10
5	10	6	10
5	10	7	10
5	10	7	10
5	10	7	10
6	11	7	11
6	11	7	11
6	11	8	11
6	11	8	11
6	11	8	11
6	11	8	11
6	11	8	11
6	11	8	12
7	11	8	12
7	12	9	12
7	12	9	12
7	12	9	13
7	14	9	13

These are inferences:  
they are guesses based  
on what we see in the  
data.

### Solution

Sample:

- The dot plots show the lengths of koi fish. The right-most peak represents the adult fish. The left-most peak, showing koi of shorter length, represents the baby fish.
- In the dot plot for Pond B the two peaks are closer together than they are in the dot plot for Pond A. So, there is less difference in length between the baby fish and the adult fish in Pond B than between the baby fish and the adult fish in Pond A.

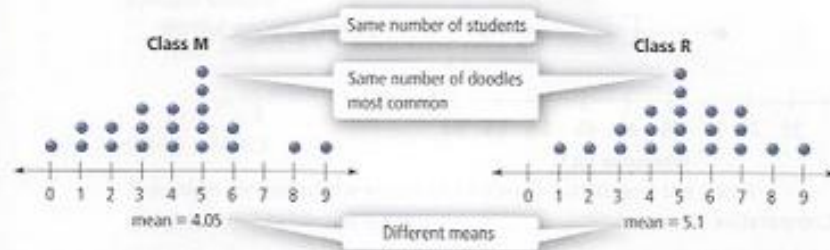
Pond A	Number of Green Frogs
2	1
4	1
5	3
6	4
7	3
8	1
9	1
10	3
11	5
12	3
14	1

A dot plot showing the number of green beads in each of 15 ponds. The horizontal axis is labeled "Pond B" and ranges from 0 to 15. The number of green beads for each pond is: Pond 5: 1, Pond 6: 2, Pond 7: 4, Pond 8: 6, Pond 9: 2, Pond 10: 4, Pond 11: 5, Pond 12: 3, Pond 13: 1.

## 15-3: Using Measures of Center

### Key Concept

You can compare two data sets by comparing their individual data values. A quicker way to make a general comparison of two data sets is to compare their measures of center.

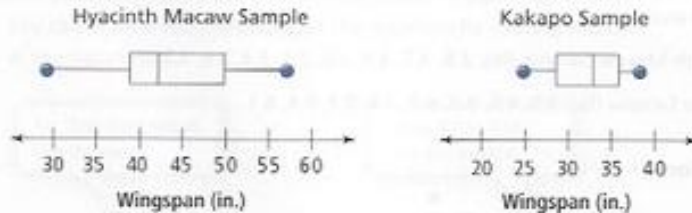


i.e. mean and median

### Part 1

#### Example Making Comparative Inferences Based on Medians

A biology student is studying two species of endangered parrots. What is the median wingspan of each sample? Make a comparative inference based on the median values.



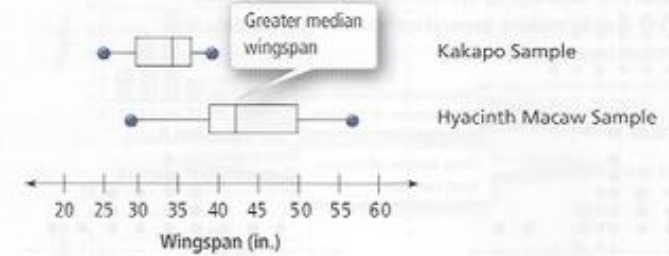
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### Part 1

#### Example continued

#### Solution

Results of a Study of Endangered Parrots



#### Comparative Inference

The Hyacinth Macaw population generally has a greater wingspan than the Kakapo population.

### Part 2

#### Example Making Comparative Inferences Based on Means

A biology student is studying two species of endangered parrots. What is the mean weight of each sample? Make a comparative inference based on the mean values.

Hyacinth Macaw Sample (lb): 2.8, 3.7, 3.9, 3.0, 3.1, 3.4, 2.9, 3.2

Kakapo Sample (lb): 3.9, 4.5, 5.3, 6.7, 7.4, 8.1, 6.4, 8.1

#### Solution

**Step 1** Calculate the mean value of each sample.

There are 8 subjects in each sample.

$$\text{mean} = \frac{\text{sum of data values}}{\text{number of data values}}$$

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## Part 2

### Solution continued

Hyacinth Macaw Sample:

$$\begin{aligned}\text{mean} &= \frac{2.8 + 3.7 + 3.9 + 3.0 + 3.1 + 3.4 + 2.9 + 3.2}{8} \\ &= \frac{26}{8} \\ &= 3.25\end{aligned}$$

Kakapo Sample:

$$\begin{aligned}\text{mean} &= \frac{3.9 + 4.5 + 5.3 + 6.7 + 7.4 + 8.1 + 6.4 + 8.1}{8} \\ &= \frac{50.4}{8} \\ &= 6.3\end{aligned}$$

The mean weight of the Hyacinth Macaw is 3.25 lb and the mean weight of the Kakapo is 6.3 lb.

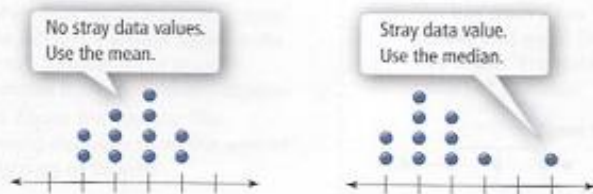
**Step 2** Make a comparative inference based on the mean values.

Sample: From the mean values, you can infer that the Hyacinth Macaws in this population are generally lighter than the Kakapos in this population.

## Part 3

### Intro

If you have different comparative inferences based on each measure of center, you can draw a conclusion about the situation by looking more closely at the data sets.



## Part 3

### Example Making Conclusions from Data

A biology student is studying two species of endangered parrots. One species can fly and the other cannot. Using the data, which species would the student conclude can fly? Explain.

HMws	Kws	HMwt	Kwt
30	25	2.8	3.9
37	26	3.7	4.5
39	30	3.9	5.3
41	33	3.0	6.7
45	34	3.1	7.4
48	35	3.4	8.1
52	36	2.9	6.4
57	37.5	3.2	8.1

### Solution

Sample:

The Hyacinth Macaw population generally has a larger wingspan and is lighter than the Kakapo population.

Using the data, the biology student might conclude that the Hyacinth Macaw is able to fly and the Kakapo is not.

Stray data values (values that are not similar to the others) will affect the mean much more than the median.

Extreme example:

Set 1: 90, 90, 90, 90, 90: mean 90, median 90

Set 2: 90, 90, 90, 90, 0: mean: 72, median 90

Remember:

- Label all answers (write down what you have found)
- Draw a box around all answers.
- You must show all work

1. Find the mean, median, mode and range of the data set: 7, 6, 4, 3, *and* 4.
2. Find the mean, median, mode and range of the data set: 8, 9, 1, 4, 5, 6, 7, 16, 6, 7.
3. A random sample is taken from two different groups of people. The age of each subject in the sample is recorded.
  - a. Find the mean age for the sample from each group.
  - b. If a person is chosen at random from each group, which group's person is likely to be younger?
  - c. Which group has a higher variability in age?

Ages of People	
Group A	Group B
18	41
19	24
38	37
31	42
38	41

4. Every five weeks, a paper supply company gives a reward to the top salesperson. The data shows the sales for the top two salespeople. Find the mean sales for each employee to decide who should get the reward.
5. The heights (in inches) of five students on a seventh grade basketball team are shown.

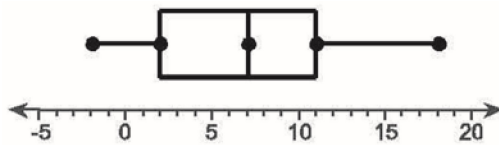
Weekly Sales	
Sophie	Xavier
\$2,236	\$2,326
\$2,033	\$2,123
\$1,932	\$2,122
\$2,635	\$2,325
\$2,235	\$2,222

67, 63, 69, 64, 67

- a. Find the mean and median height of the players.
- b. The tallest student is replaced at center by a new student who is 79 inches tall. Calculate the new mean and median heights for the team.
- c. What did you notice about the changes in the mean and median heights when the new student was added?

**Practice  
15-1****Statistical Measures**

1. Find the mean and median of the data set 5, 8, 5, 9, 1, and 2.
2. Find the mean and median of the data set 9, 8, 7, 4, and 5.
3. What is the interquartile range of this box plot?



4. The data set 55, 65, 40, 40, 30, 50, 64, 45, 40, and 41 shows the admission price (in dollars) for one-day tickets to 10 theme parks in the United States. What is the interquartile range of the data values?

5. A random sample is taken from two different groups of people. The age of each subject in the sample is recorded.

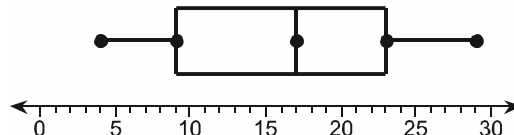
- a) Find the mean age for the sample from each group.
- b) If a person is chosen at random from each group, which group's person is likely to be younger?

Ages of People	
Sample from Group A	Sample from Group B
18	41
19	24
38	37
31	42
38	41

6. The data set shows the time (in seconds) it takes a random sampling of five race cars to complete one lap of a race track.
  - a) Find the range and interquartile range of the data set 25.4, 26.8, 29.9, 31.3, and 34.4.
  - b) Describe the variability of the data.
    - ☐ A. The times are far apart, so there is high variability.
    - ☐ B. The times are all the same, so there is no variability.
    - ☐ C. The times are close together, so there is low variability.
7. a) **Writing** Find the mean and median of the data set 1,199, 958, 1,240, 1,094, 1,153, and 957.
  - b) Describe a situation you could model using this data set. Interpret the mean and median in the context of your situation.



8. a) **Reasoning** Use the range and interquartile range to describe the variability of the data set 8, -4, -8, -2, -3, 7, 9, 16, and -9.
- b) For this data set, is the range or the interquartile range a better measure of the variability?
- ☐ A. The interquartile range, because the range is too large.
  - ☐ B. The range, because the value 16 is a stray data value.
  - ☐ C. The interquartile range, because the value 16 is a stray data value.
  - ☐ D. The range, because the interquartile range is too small.
9. **Error Analysis** Your friend incorrectly says that the interquartile range of this box plot is 12.



- a) What is the correct interquartile range?
- b) What mistake might your friend have made?
- ☐ A. Your friend found the difference between the third quartile and the median.
  - ☐ B. Your friend found the difference between the maximum and the minimum.
  - ☐ C. Your friend found the difference between the median and the first quartile.
  - ☐ D. Your friend found the difference between the maximum and the median.
10. **Hotel Rooms** Before going on vacation, Leo randomly samples the cost of hotel rooms at two different hotels. Which hotel is Leo likely to choose?
- ☐ A. Leo is likely to choose Hotel S.
  - ☐ B. Leo is likely to choose Hotel T.
  - ☐ C. It is impossible to compare the two hotels.

Cost per Night (\$)	
Hotel S	Hotel T
108	107
137	129
178	182
238	213
407	241

11. **Mental Math** Find the interquartile range of the data set 32.6, 98.5, 16.6, 22.4, 99.8, 72.6, 68.2, 51.8, and 49.3.

12. **Estimation** Simple random samples are taken from two groups of mice in a laboratory to study their weights.

Weight of Mice in Laboratory (grams)						
Data Set X	104	147	127	117	157	174
Data Set Y	176	74	143	242	205	158

- a) Compare the ranges of Data Set X and Data Set Y. Fill in the answer line to complete your choice.
- ☐ A. The range of Data Set X is about \_\_\_\_\_ times greater than the range of Data Set Y.
  - ☐ B. The range of Data Set Y is about \_\_\_\_\_ times greater than the range of Data Set X.
  - ☐ C. The ranges of Data Set X and Data Set Y are equal.
  - ☐ D. It is impossible to compare the ranges of the data sets.
- b) What can you conclude about Data Set Y?
- ☐ A. The weights of the mice in Data Set Y have less variability than the mice in Data Set X.
  - ☐ B. Data Set Y contains more mice than Data Set X.
  - ☐ C. The weights of the mice in Data Set Y have greater variability than the mice in Data Set X.
  - ☐ D. Data Set Y contains fewer mice than Data Set X.

13. Compute the mean and median of the following data set.

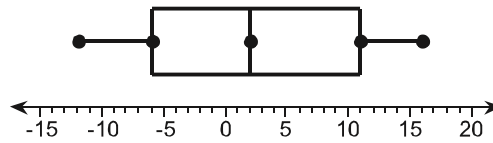
150    122    168    196    104    189    118

14. **Think About the Process** The annual salaries of the employees of a local cable network office are shown below.

\$26,800    \$12,200    \$34,500    \$22,900    \$28,800    \$19,700

- a) What is the first step when finding the mean of a data set?
- ☐ A. Find the sum of the data values.
  - ☐ B. Write the data values in order from least to greatest.
  - ☐ C. Subtract the least data value from the greatest data value.
  - ☐ D. Choose the middle value of the data set.
- b) The mean annual salary is \$\_\_\_\_\_.
- c) The median annual salary is \$\_\_\_\_\_.

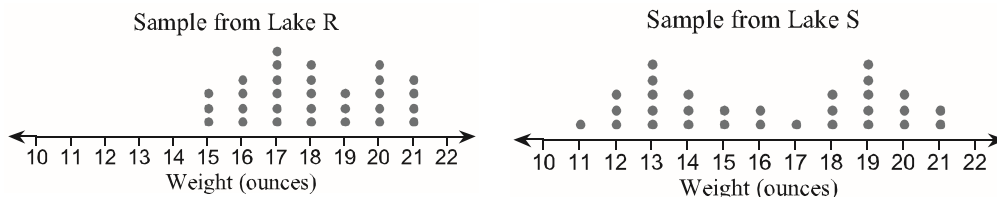
15. Think About the Process



- a) What is always true about the interquartile range of a data set? Select all that apply.
- ☐ A. The interquartile range is less than the range.
  - ☐ B. The interquartile range contains about 50% of the data set.
  - ☐ C. The interquartile range contains the median of the data set.
  - ☐ D. The interquartile range is greater than the range.
- b) The interquartile range is \_\_\_\_\_.

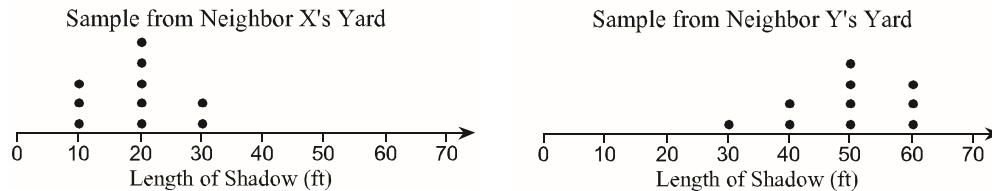
**Practice  
15-2*****Multiple Populations and Inferences***

1. A local appliance store is reviewing the performances of its junior, mid-level, and senior sales associates over the past month. For which question should the sales associates be considered as one population to find the answer?  
☐ A. Which group of associates had the greatest combined sales?  
☐ B. How many associates from each group reached their sales quotas?  
☐ C. Who made the least sales of all associates?  
☐ D. Who made the most sales from each group?
2. You are researching the different cell phones offered by 5 major companies. For which question should you consider the cell phones as multiple populations to find the answer?  
☐ A. Which is the largest cell phone overall?  
☐ B. What is the total number of different cell phones the 5 companies offer?  
☐ C. How many of the cell phones can connect to a computer?  
☐ D. Which company offers the greatest number of different cell phones?
3. Victoria is researching the vehicles at a local dealership. The dealership sells cars, trucks, and vans. How many populations should she use to answer the following question: How many vehicles does the dealership sell each month?
4. Ivan is researching the television viewing habits of the people at his school. Which population(s) should he use to find out if a higher percentage of students or teachers watch television every night?  
☐ A. the population of Ivan's grade  
☐ B. the population of teachers, the population of boys, and the population of girls  
☐ C. the population of teachers and the population of students  
☐ D. the population of all people at the school
5. The dot plots show the weights of a random sample of fish from two lakes.



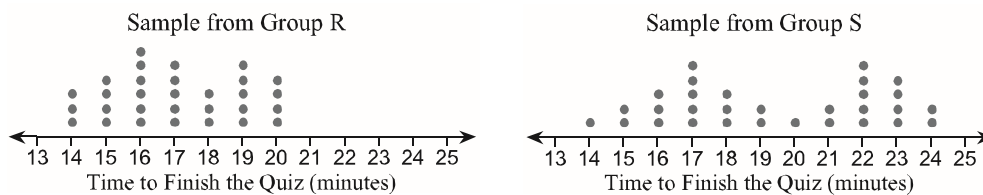
- a) Which describes the variability of the samples?
- ☐ A. The variability of the weights of the fish in Lake R is greater than the variability of the weights of the fish in Lake S.
  - ☐ B. The variability of the weights of the fish in the two lakes are about the same.
  - ☐ C. The variability of the weights of the fish in Lake S is greater than the variability of the weights of the fish in Lake R.
- b) Which makes a comparative inference about the fish in the two lakes?
- ☐ A. There is about the same difference in weight between small and large fish in both lakes.
  - ☐ B. There is less difference in weight between small and large fish in Lake S than between small and large fish in Lake R.
  - ☐ C. There is less difference in weight between small and large fish in Lake R than between small and large fish in Lake S.

6. a) The dot plots show samples of the lengths of the shadows cast by elm trees in two of your neighbors' yards at the same time of day. Find the median of each sample.



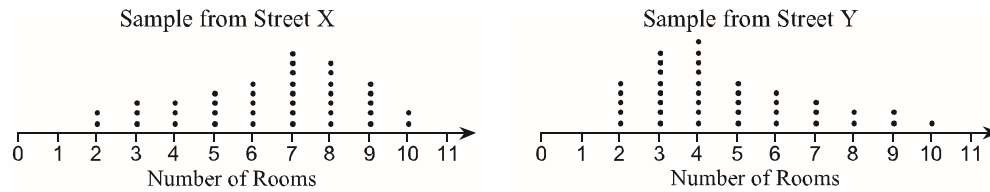
- b) Which makes a comparative inference about the elm trees in the two yards?
- ☐ A. There are more trees in Neighbor Y's yard than in Neighbor X's yard.
  - ☐ B. Most of the elm trees in Neighbor X's yard are older than the elm trees in Neighbor Y's yard.
  - ☐ C. There are more trees in Neighbor X's yard than in Neighbor Y's yard.
  - ☐ D. Most of the elm trees in Neighbor Y's yard are older than the elm trees in Neighbor X's yard.
7. **Writing** You are researching the different animals in three pet stores.
- a) For which question should you consider the animals in each store as multiple populations to find the answer?
- ☐ A. What is the total number of animals in the three stores?
  - ☐ B. Which store has the most animals overall?
  - ☐ C. What is the largest animal in any of the stores?
  - ☐ D. What is the most expensive pet in any store?
- b) Can one population in a study be considered more than one population in another study? Explain.

- 8. Reasoning** A researcher gave the same quiz to two groups. The dot plots show the times it took the people in each group to finish the quiz.



- a) Which describes the variability of the samples?
- ☐ A. The variability of the times for Group S is greater than the variability of the times for Group R.
  - ☐ B. The variability of the times for the two groups are about the same.
  - ☐ C. The variability of the times for Group R is greater than the variability of the times for Group S.
- b) Which makes a comparative inference about the people in the two groups?
- ☐ A. There is about the same difference in time among the people in both groups.
  - ☐ B. There is less difference in time among the people in Group R than among the people in Group S.
  - ☐ C. There is less difference in time among the people in Group S than among the people in Group R.
- c) Give a possible reason for the two peaks in each dot plot. Explain your reasoning.
- 9. Error Analysis** Manny is researching the heights of the athletes at his school. His school has 5 different sports teams for girls and 5 different sports teams for boys. Manny wants to know whether the baseball or football team has a greater mean height. His friend incorrectly stated that he should use 1 population to answer the question.
- a) How many populations should he use?
- b) What mistake might his friend have made?
- ☐ A. His friend included a population that is not required to answer the question.
  - ☐ B. His friend included a greater number of populations than the total number of sports teams.
  - ☐ C. His friend left out a population that is required to answer the question.

- 10. Houses** The dot plots show samples of the number of rooms in homes on two adjacent streets.



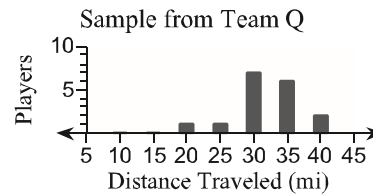
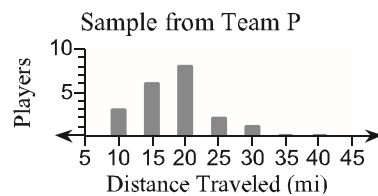
- a) Find the median of each sample.
- b) Which makes a comparative inference about the homes on the two streets?
- ☐ A. There are more homes on Street X than on Street Y.
  - ☐ B. There are more homes on Street Y than on Street X.
  - ☐ C. Most of the homes on Street X are larger than the homes on Street Y.
  - ☐ D. Most of the homes on Street Y are larger than the homes on Street X.
- 11. Open-Ended** You are studying the different types of trees in a city park.
- a) For which question should you consider the trees as multiple populations to find the answer?
- ☐ A. What is the height of the tallest tree overall?
  - ☐ B. How many of the trees have a trunk at least 3 feet wide?
  - ☐ C. Does the number of trees vary among the different types?
  - ☐ D. What is the total number of trees in the park?
- b) Write three more questions for which you should consider the trees as multiple populations to find the answer.
- 12.** Anita is choosing universities to apply to. Some are private schools and some are state schools. She writes a list of questions.
- a) For which question should she consider the universities as one population to find the answer?
- ☐ A. Do the private or state schools have a lesser mean tuition?
  - ☐ B. Which universities have the least amount of tuition for each type?
  - ☐ C. Which university offers the most scholarships?
  - ☐ D. Which type offers more scholarships?
- b) Explain why she might want to ask each of these questions.
- 13.** A local movie store sells 8 different categories of movies.
- a) How many populations should you use to answer the following question?
- Which is the least expensive movie in the store?
- b) Give two more questions you could ask about the movies in the store. Explain how many populations you should use to answer each.

- 14. Think About the Process** David is studying the mountains on each of the seven continents in the world. He asks the following question.

How many mountains are there in the world?

- a) Select each factor he should consider when deciding which population(s) to use.
- ☐ A. Use only one population.
  - ☐ B. Use all populations that are required to answer the question.
  - ☐ C. Do not use any populations that are not required to answer the question.
  - ☐ D. Use as many populations as you can think of.
- b) List or describe the population(s) he should use.
- ☐ A. The populations of mountain ranges on each of the seven continents
  - ☐ B. The populations of mountain ranges in North America and in Asia
  - ☐ C. The populations of mountain ranges in each of North America, Asia, and Europe
  - ☐ D. The population of all mountain ranges in the world

- 15. Think About the Process** The bar graphs show samples of the distances players on two soccer teams had to travel to get to the soccer field for their game.

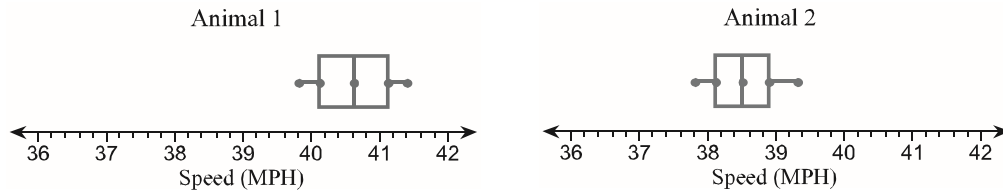


- a) What is the first step in finding the median of each sample?
- ☐ A. Find the sum of the values in the sample.
  - ☐ B. Arrange the values in the sample in order from least to greatest.
  - ☐ C. Find the number of values in the sample.
  - ☐ D. Divide the number of values in the sample by 2.
- b) The median of the sample from Team P is \_\_\_\_\_ mi.
- c) The median of the sample from Team Q is \_\_\_\_\_ mi.
- d) Make a comparative inference about the distance traveled by the players on the two teams.
- ☐ A. Most of the players on Team Q live closer to the field than the players on Team P.
  - ☐ B. There are more players on Team P than on Team Q.
  - ☐ C. There are more players on Team Q than on Team P.
  - ☐ D. Most of the players on Team P live closer to the field than the players on Team Q.

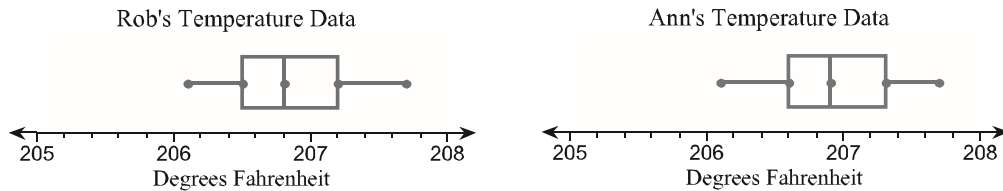


**Practice  
15-3****Using Measures of Center**

1. The box plots display how fast two different animals can run. Compare the median of each box plot. Which of the following is the best description of the medians?



- ☐ A. The median for Animal 2 is greater.
- ☐ B. The medians are about the same.
- ☐ C. The median for Animal 1 is greater.
2. Water boils at different temperatures based on its elevation above sea level. Rob and Ann are in different cities. They both boil water in a number of pots. Each person records the water temperature just as the water starts to boil. The higher the elevation is the lower the boiling point is and the lower the elevation is the higher the boiling point is. They use box plots to display their data. Compare the medians of the box plots.



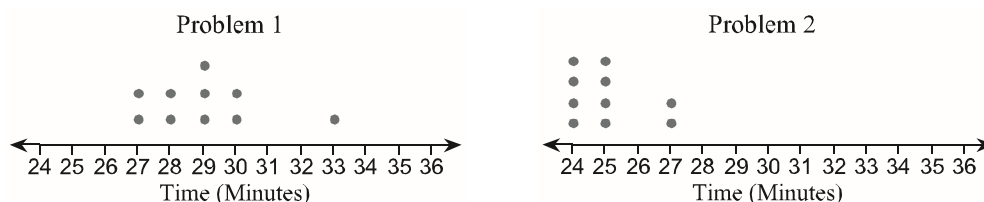
- a) Which of the following is the best description of the medians?
- ☐ A. The median from Rob's data is greater.
- ☐ B. The median from Ann's data is greater.
- ☐ C. The medians are about the same.
- b) Which of the following is the best inference based on the median values?
- ☐ A. Ann is at a greater elevation.
- ☐ B. Rob and Ann are at about the same elevation.
- ☐ C. Rob is at a greater elevation.

3. An experiment compares the life spans of two brands of light bulbs. Twelve bulbs of each type were tested until failure. The results are shown for the two types of light bulbs in hours.

Bulb A: HotLight Time, in hours		
967	1,255	1,474
1,451	1,520	1,033
963	1,363	1,437
1,393	1,050	1,512

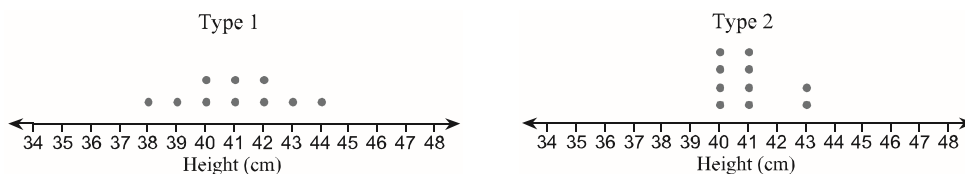
Bulb B: BrightBulb Time, in hours		
1,147	1,418	997
931	1,452	1,096
941	1,003	1,477
1,147	1,094	1,545

- a) Find the mean time of each light bulb.
- b) Which of the following is the correct description of the populations?
- ☐ A. HotLight generally has the greater lifespan.
  - ☐ B. BrightBulb generally has the greater lifespan.
  - ☐ C. The two bulbs generally have the same lifespan.
4. The dot plots show the amount of time it takes each person, in a random sample, to complete two similar problems.



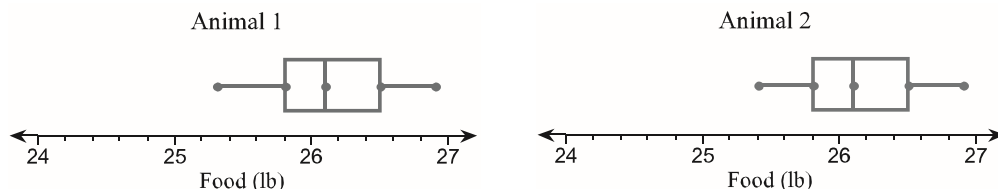
- a) What is the mean time for each problem?
- b) Which of the following is a correct inference based on the mean values?
- ☐ A. Problem 2 is more challenging than Problem 1.
  - ☐ B. Problem 1 is more challenging than Problem 2.
  - ☐ C. The problems are equally challenging.
5. A study is done to compare the fuel efficiency of cars. Cars in group 1 generally get about 32 miles per gallon. Cars in group 2 generally get about 44 miles per gallon.
- a) Which of the following is a correct inference based on the mean values?
- ☐ A. The cars in group 1 and group 2 are equally fuel efficient.
  - ☐ B. The cars in group 2 generally are more fuel efficient.
  - ☐ C. The cars in group 1 generally are more fuel efficient.
- b) Which of the following is another conclusion that a person might make from the mean values?
- ☐ A. The cars in group 1 are smaller.
  - ☐ B. The cars in each group are about the same size.
  - ☐ C. The cars in group 2 are smaller.

6. A family is looking for a new home. The family is comparing homes in two towns. The median home price of Town 1 is \$200,000. In Town 2 the median home price is \$200,000.
- a) Which of the following is a correct inference based on the median values?
- ☐ A. The homes in Town 1 are more expensive.
  - ☐ B. The homes in Town 2 are more expensive.
  - ☐ C. The homes in Town 1 and Town 2 are similarly priced.
- b) Which of the following is another conclusion that a person might make from the median values?
- ☐ A. The homes in Town 1 are probably greater in size.
  - ☐ B. The homes in each town are about the same size.
  - ☐ C. The homes in Town 2 are probably greater in size.
7. **Writing** A group is comparing the number of people that go to two different chain stores. The mean number of people that go to Store 1 each day is 894. The mean number of people that go to Store 2 each day is 1,030.
- a) Which of the following is a correct inference based on the mean values?
- ☐ A. Each store generally has the same amount of customers each day.
  - ☐ B. Store 1 generally has more customers each day.
  - ☐ C. Store 2 generally has more customers each day.
- b) Which of the following is another conclusion that a person might make?
- ☐ A. The prices at Store 2 are better.
  - ☐ B. The prices are about the same at each store.
  - ☐ C. The prices at Store 1 are better.
- c) Write a different conclusion based on the mean values.
8. **Reasoning** There are two types of plants in a greenhouse. The dot plots show the heights of 20 plants, 10 of Type 1 and 10 of Type 2. The plants are given the same amount of food and water.

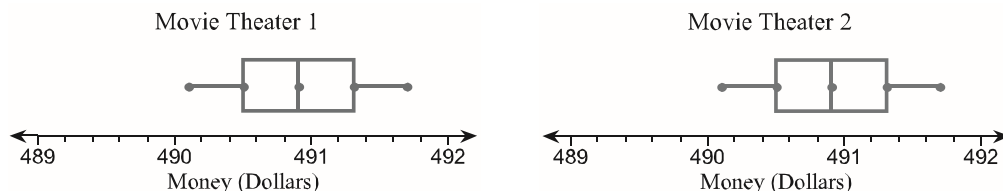


- a) What is the mean height for each type of plant?
- b) Which of the following is a correct inference based on the mean values?
- ☐ A. The two types were planted at the same time.
  - ☐ B. Type 1 was planted first.
  - ☐ C. Type 2 was planted first.
- c) Explain why you would want to use a dot plot instead of a box plot to find the mean.

- 9. Error Analysis** Elise is asked to compare the medians of the two sets of data. The data compares the amount of food two animals eat over a period of days. Elise says, "The medians are the same, so Animal 2 generally eats more food."



- a) Which of the following is the best description of the medians?
- ☐ A. The median for Animal 1 is greater.
  - ☐ B. The medians are the same.
  - ☐ C. The median for Animal 2 is greater.
- b) Which of the following is the best inference based on the median values?
- ☐ A. Animal 2 generally eats more food.
  - ☐ B. Animal 1 generally eats more food.
  - ☐ C. The animals generally eat the same amount of food.
- c) What error might Elise have made?
- ☐ A. Elise's inference was incorrect.
  - ☐ B. Elise's description of the medians was incorrect.
  - ☐ C. Elise's inference and description of the medians was incorrect.
- 10. Movie Theaters** There are two movie theaters in a town. A student wants to compare the amount of money each theater makes over a period of days. The box plots show the data for the amount of money each theater makes over a period of days. Compare the median of each box plot.



Which of the following is the best description of the medians?

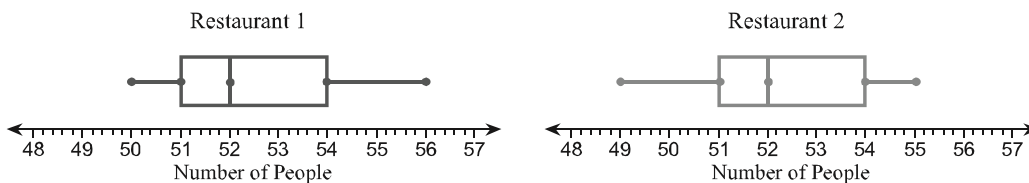
- ☐ A. The median for Movie Theater 2 is greater.
- ☐ B. The median for Movie Theater 1 is greater.
- ☐ C. The medians are the same.

- 11. Mental Math** A student is studying two different breeds of dogs. The following are the weights of 6 of each breed of dog.

Dog Breed 1 Weight (lb)		
61	59	48
54	50	46

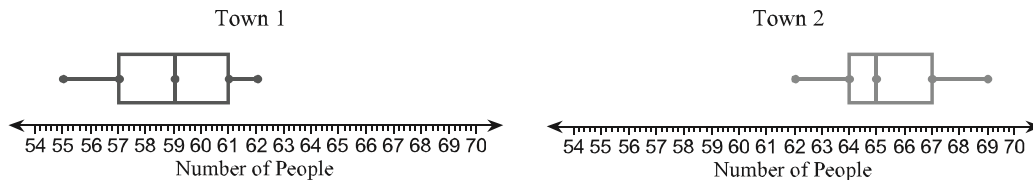
Dog Breed 2 Weight (lb)		
44	38	43
46	45	36

- a) Find the mean weight of each population.
- b) Which of the following is a correct inference based on the mean values?
- ☐ A. Dog Breed 2 generally weighs more than Dog Breed 1.
  - ☐ B. Dog Breed 1 generally weighs more than Dog Breed 2.
  - ☐ C. Dog Breed 1 and Dog Breed 2 generally weigh about the same.
- c) Explain how you could use mental math to compare the means of the two types of dogs.
- 12.** The box plots show the number of people that went to two restaurants for lunch over a few days.

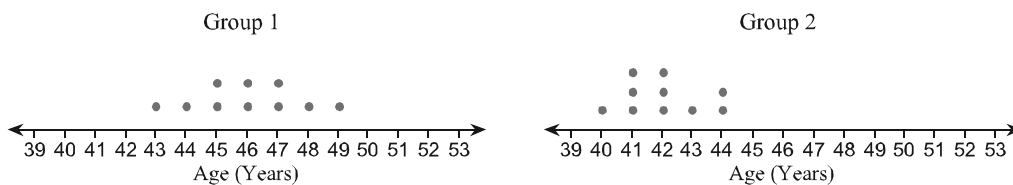


- a) Which of the following is the best description of the medians?
- ☐ A. The median for Restaurant 1 is greater.
  - ☐ B. The median for Restaurant 2 is greater.
  - ☐ C. The medians are the same.
- b) Which of the following is the best inference based on the median values?
- ☐ A. More people generally eat at Restaurant 1 for lunch.
  - ☐ B. The same amount of people generally eat at each restaurant for lunch.
  - ☐ C. More people generally eat at Restaurant 2 for lunch.
- c) When making an inference, does the inference always apply to the entire population of the two samples being compared? Explain.

13. The box plots show the number of people that go to town meetings in two towns. The data is over the period of one year.



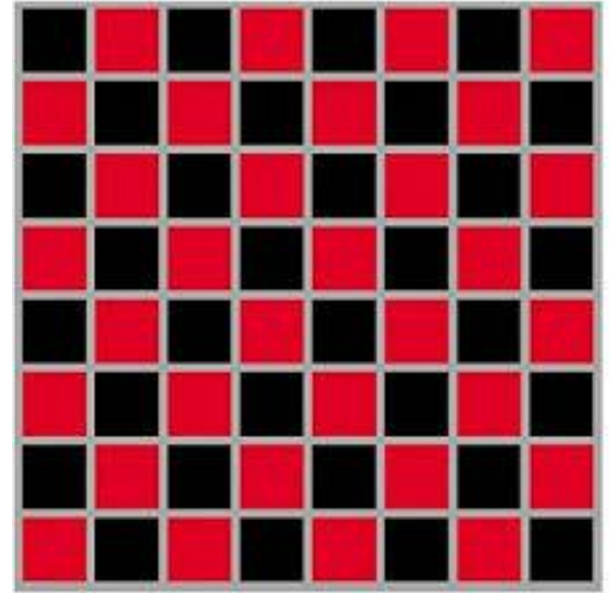
- a) Which of the following is the best inference based on the median values?
- ☐ A. More people generally go to the meetings in Town 1.
  - ☐ B. The same amount of people in each town generally go to the meetings.
  - ☐ C. More people generally go to the meetings in Town 2.
- b) Which of the following is another conclusion that a person might make from the median values?
- ☐ A. The meetings in Town 2 are more important.
  - ☐ B. The meetings in both towns are equally important.
  - ☐ C. The meetings in Town 1 are more important.
14. **Think About the Process** The following dot plots show the ages of two groups of tourists.



- a) What is the first step to finding the mean age for each group?
- ☐ A. The first step should be to average the minimum and maximum values that have dots.
  - ☐ B. The first step should be to count the ages that have dots and just average those numbers.
  - ☐ C. The first step should be to count the number of dots for each age.
  - ☐ D. The first step should be to find the middle number of all the dots.
- b) The mean age for Group 1 is \_\_\_\_\_ years.
- c) The mean age for Group 2 is \_\_\_\_\_ years.
- d) Which is a correct inference based on the mean values?
- ☐ A. Group 1 is generally older than Group 2.
  - ☐ B. Group 2 is generally older than Group 1.
  - ☐ C. The groups are generally the same age.

- 15. Think About the Process** A family is comparing natural stones used for kitchen countertops. They go to various stores to compare the price of two types of stones. The mean price per square foot of Stone 1 is \$78 and the mean price per square foot for Stone 2 is \$118.
- a) Which is a correct inference based on the mean values?
- ☐ A. Stone 2 is generally more expensive than Stone 1.
  - ☐ B. Stone 1 is generally more expensive than Stone 2.
  - ☐ C. Stone 1 and Stone 2 generally cost the same.
- b) Use the mean price to draw a conclusion. Do you think the stone with the greater mean price is more or less rare than the other stone?
- c) Which is a reasonable conclusion?
- ☐ A. Stone 2 is a rarer stone.
  - ☐ B. Stone 1 is a rarer stone.
  - ☐ C. The rareness of each stone is the same.

How many squares are there on a standard checkerboard? The answer is not one, nor is it sixty-four. There are squares of many different sizes. Can you find the number of all of the different squares? Does a pattern exist that might help you solve this problem?





## Week 5 – The Fibonacci Series and Other Oddities (5/18/20 –5/22/20)

Note: This assignment will not be collected. Please look it over before you decide to not do it. You may actually enjoy this.

**Goal:** To investigate some of the things that mathematicians think about when they have too much free time. Frequently, these things turn out to be very useful and/or interesting to many.

Office Hour Schedule	
Monday	Patterns and the Fibonacci Sequence
Tuesday	Fibonacci Spiral
Wednesday	Kahoot!
Thursday	Hexaflexagon
Friday	Tetraflexagon

Contact		
Office hours by Email:	Mon – Fri: 8:00 AM – 3:30 PM mdibley@tusd.net	
Office hours by video:	Mon – Fri: 10:00 – 11:00 AM <a href="https://zoom.us/j/312003066">https://zoom.us/j/312003066</a>  Meeting ID: 312 003 066 Password: 805373	Mon – Fri: 3:00 – 4:00 PM <a href="https://zoom.us/j/218432703">https://zoom.us/j/218432703</a>  Meeting ID: 218 432 703 Password: 672048

How to get an assignment:	
Digital Option	non-Digital Option
<ul style="list-style-type: none"> <li>Videos and Notes will all be uploaded to digits on (or before) Monday, May 18.</li> <li>You will not need to submit anything.</li> </ul>	<ul style="list-style-type: none"> <li>Lessons will be provided in a paper format.</li> <li>A packet must be picked up from the George Kelly office on Friday May 8. This is the last pick-up day current scheduled.</li> <li>There is no drop-off day for this assignment.</li> </ul>

### Digital Option

This is the only file for this week. Scroll down for daily questions, further instructions, project templates, and the logic problem.

- Monday: Patterns
  - Attempt to discover the patterns in the numerical sequences (see file).
  - Video: “The Fibonacci Sequence and Rabbits” (<https://youtu.be/sjQIW6cH3Ko>)
- Tuesday: Fibonacci Sequence
  - Video: “Doodling in Math: Spirals, Fibonacci, and Being a Plant” (<https://youtu.be/ahXIMUKSXX0>)
  - Create a Fibonacci Spiral (graph paper included in file)
- Wednesday: Roman Numerals
  - Audio: “18 Wheels on a Big Rig” (<https://youtu.be/aVRWRQJZoE>)
  - Join me for a kahoot! lesson on Roman numerals. There will be a new game starting at 10, 10:30, 3 and 3:30.
- Thursday: Hexaflexagon
  - Video background: “Hexaflexagons” (<https://youtu.be/VIVlegSt81k>)
  - Video instructions: “How to Make a Hexaflexagon: the Definitive Guide” (<https://youtu.be/Svq2Kscmmwc>)
  - Written instructions and a handy template included in the file.
- Friday: Tetraflexagon
  - Video: “Tetraflexagons” (<https://youtu.be/7H4IDi79YY8>) [Once the instructions are over, you might find the rest of this a little boring. It is okay to stop]
  - Written instructions are included in the file.
- Bonus Logic Problem: Magic Square

## MONDAY - Patterns

For each sequence of numbers, describe the pattern (write a sentence) and then determine the next three numbers in the sequence.

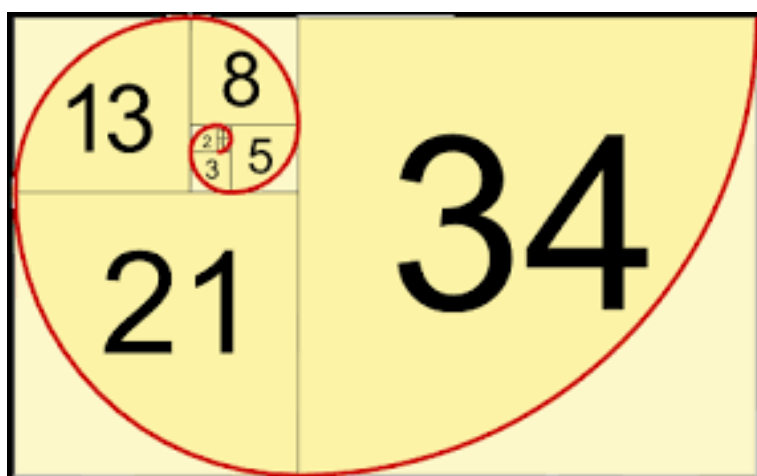
- A. 7, 11, 15 ...
- B. 4, 12, 36 ...
- C. 8, 4, 2, 1 ...
- D. 1, 3, 6, 10, ...
- E. 1, 4, 9, 16 ...
- F. 1, 1, 2, 3, 5, 8, ...

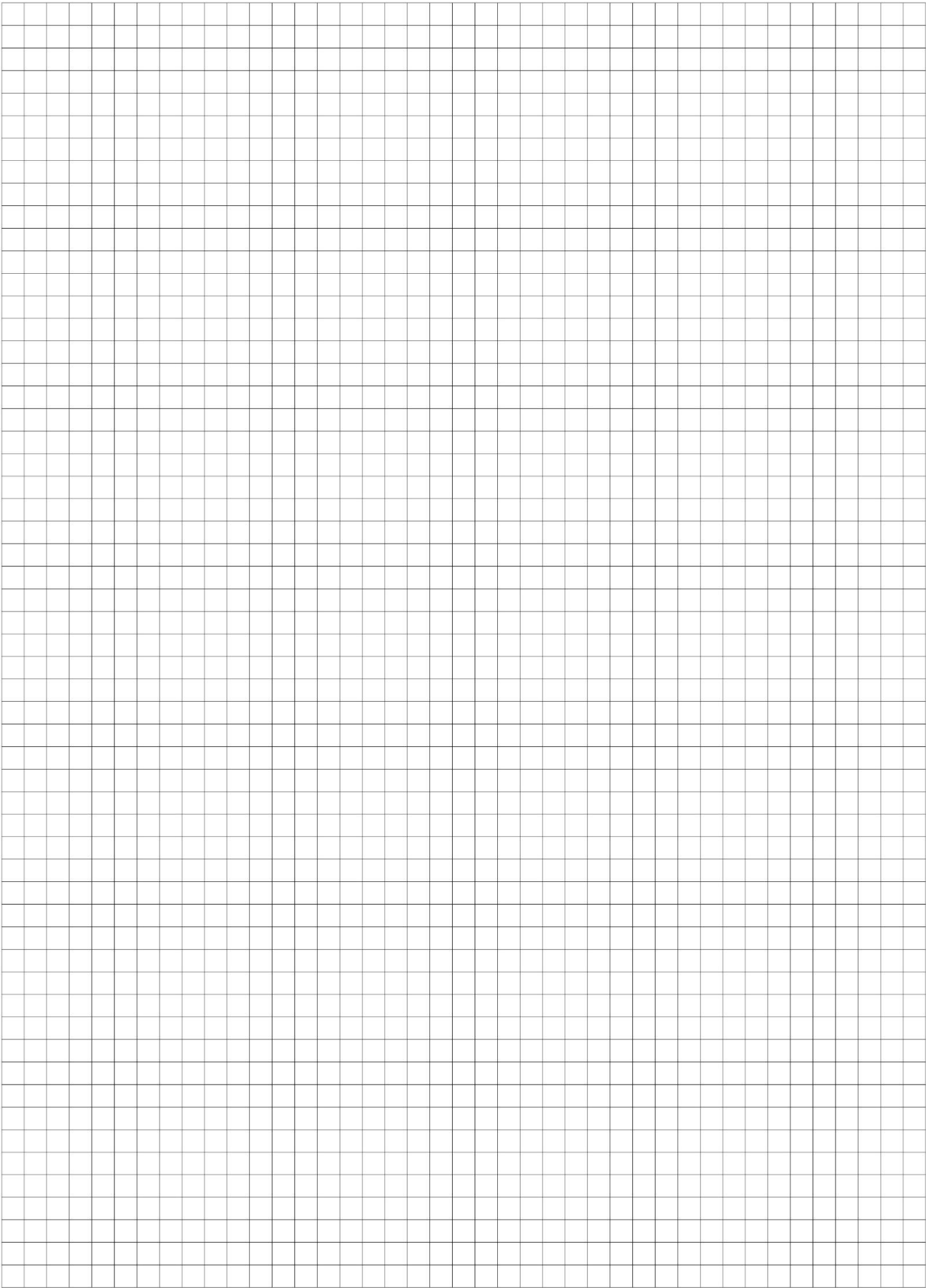
(see the end of the file for solutions)

## TUESDAY – Fibonacci Spiral

(Note, a sheet of graph paper is included on the next page)

- Starting from the upper left corner, count over about 15 squares.
- Count down about 10 squares, and draw a 1x1 box.
- To the left of this box, draw another 1x1 box that is in contact with the first box.
- Below this 1x2 rectangle draw a 2x2 box that is in contact with it.
- To the right of this new rectangle draw another square box that has the same side length as the rectangle.
- Continue adding squares in a spiral fashion.
- Draw a smooth curve though the boxes, beginning with the smallest boxes.





## WEDNESDAY – Roman Numerals

If you have a phone or computer, join me for Kahoot! lesson on Roman numerals. There will be a new Kahoot! starting at 10, 10:30, 3 and 3:30.

The next page has a guide on how to convert Roman numerals. Read the guide and then try the following problems.

<b>I</b>	<b>1</b>
<b>V</b>	<b>5</b>
<b>X</b>	<b>10</b>
<b>L</b>	<b>50</b>
<b>C</b>	<b>100</b>
<b>D</b>	<b>500</b>
<b>M</b>	<b>1000</b>

Got It? Try these.

Convert the following numbers to Roman numerals

1. 38                      2. 89                      3. 333                      4. 2,143                      5. 494

Convert the following to Hindu-Arabic numerals

6. XXVIII              7. CLXXXVII              8. CDXXXVI              9. MMXCVII              10. CMXCIX

## A Very Brief History of Roman Numerals

Several hypotheses exist as to the origin of Roman numerals. The common thread through all of them is that the numerals were developed primarily as a counting system for commerce, with the most popular theory being that they began as notches on tally sticks. A single notch represented “one.” Every fifth notch was double cut to form a “V” shape and every tenth notch was double crossed to form an “X.” The Romans later transferred this tally system to writing and designated Roman letters to certain values.

A defining and extremely limiting trait of the Roman numeral system is that it lacks a character to designate the number 0. The system also has no way to represent negative or decimal numbers. This all goes back to the fact that Roman numerals were developed primarily to count and keep track of things for commerce. Consequently, higher-level math was and is pretty much impossible with this system.

Even after the decline of the Roman Empire, their numerals continued to be used throughout Europe, up through the Middle Ages. It wasn’t until the 14th century that Hindu-Arabic numerals (these are the numerals that we use today) replaced Roman numerals. Even after the rise of the former, the Roman system continued on as a sort of antiquated shout out to things old and classic.

### How to Read Roman Numerals: Know the symbols and their values.

- Reading Roman numerals is pretty easy once you understand the basic symbols and their corresponding values. There are seven basic symbols. With these seven symbols you can fashion pretty much any number (with the exception of exotic numbers like negatives, decimals, etc.).
- When one or more numeral is used to form a number, the value of each symbol is (generally) added together from left to right.
- The letters are arranged from left to right in descending order of value to form a number:

$$II = 2$$

$$XXX (10 + 10 + 10) = 30$$

$$LII (50 + 1 + 1) = 52$$

$$MMLVII (1,000 + 1,000 + 50 + 5 + 1 + 1) = 2,057$$

- In some instances, a lower numeral placed in front of a larger numeral indicates that the lower numeral should be subtracted from the larger.
- The subtraction rule (usually) kicks in when you’re getting close to a number that has a unique symbol. Take 4, for instance. It’s just one number away from 5, which has its own symbol (“V”). Instead of writing out four “I”s, you’d simply write “IV,” which indicates that you subtract 1 from 5 to get 4.
- How about 9? It’s just one digit away from 10, which has its own symbol (“X”). So instead of “VIII” (5+1+1+1+1), you’d write “IX” (1 subtracted from 10).
- And what about 40? It’s just 10 away from 50 which has its own symbol (“L”), so instead of writing out XXXX, you’d simply put XL (10 subtracted from 50). Just remember, when you see a lower value in front of a higher value, this rule kicks in.
- Some more examples of the subtraction rule in action:

$$29 = XXIX (10 + 10 + (10 - 1))$$

$$399 = CCCXCIX (100 + 100 + 100 + (100 - 10) + (10 - 1))$$

$$444 = CDXLIV ((500 - 100) + (50 - 10) + (5 - 1))$$

## THURSDAY – Hexaflexagons

Follow the directions as shown on the template.

Helpful (?) Hints:

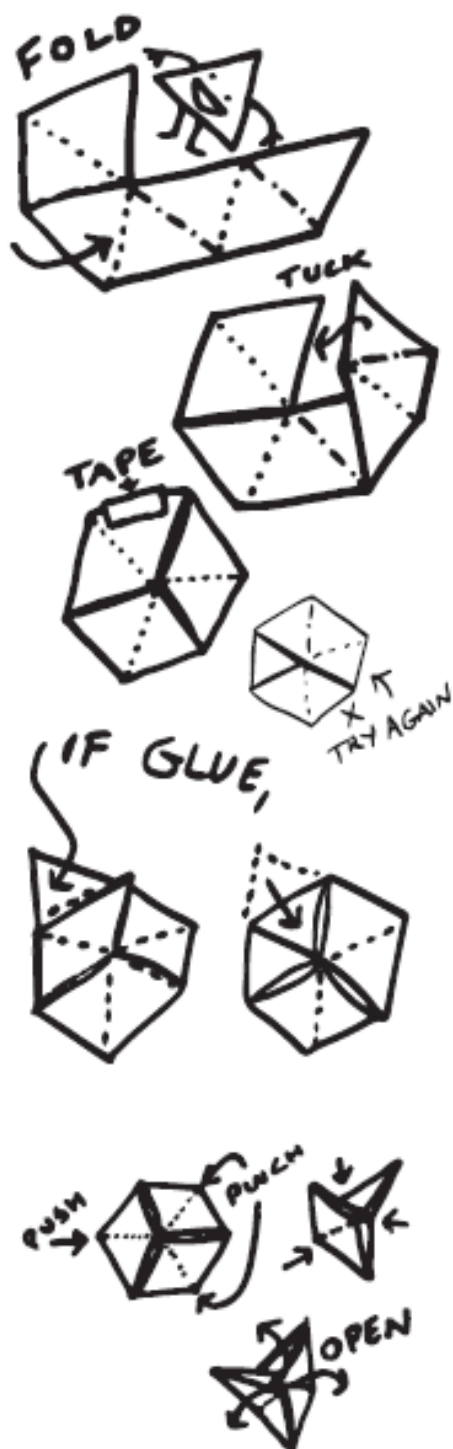
1. Once the shape is cut out, fold the shape back and forth into equilateral triangles. It will be easier to use later, if all of the dashed lines are creased now.
2. Note that the type of dashed line indicates whether the fold should be a “mountain” fold or a “valley” fold.

Mountain fold:    

Valley Fold:    

3. For a “mountain” fold, bend the paper away from the dashed line so that it sits on a ridge once the paper is folded.
4. For a “valley” fold bend both sides towards the dashed line so that it sits on a valley floor once the paper is folded.
5. The tricky part is the tuck. Pay attention to the picture.
6. Once complete, there should be three flaps, equally spaced around the hexaflexagon. If not, try again.

# HEXAFLEXAGON



extra triangle to be glued to the first,  
skip if using tape

Extra-robust version:  
fold down the middle first

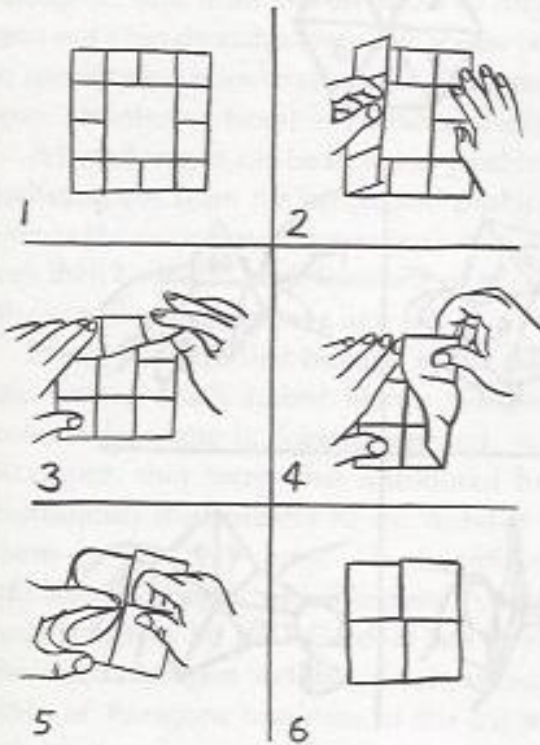


## FRIDAY - Tetraflexagons

Begin with a square piece of paper. Fold it in half and in half again. Turn it 90° and then fold it in half and in half again. When you unfold the paper it should look like a 4x4 grid. Cut out the middle four squares.



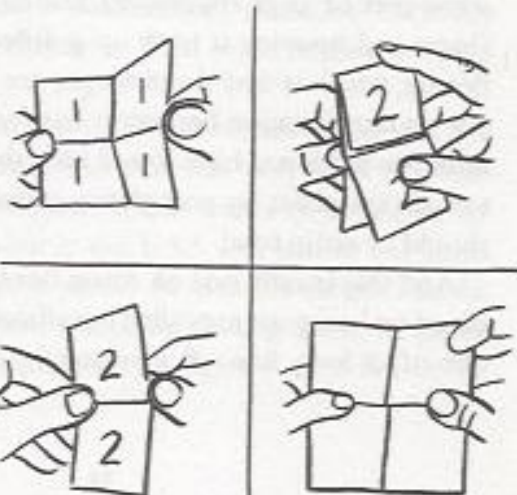
### Folding a Tetraflexagon



1. Start with a 'ring' of twelve squares.
2. Fold the four left-hand squares over.
3. Fold the top three squares down.
4. Fold the right-hand three squares in.
5. Fold the bottom two up, tucking them under the original four you folded in from the left.
6. The result should be a symmetrical square.

### Flexing a Tetraflexagon

1. Fold the tetraflexagon over in half, hiding the first face.

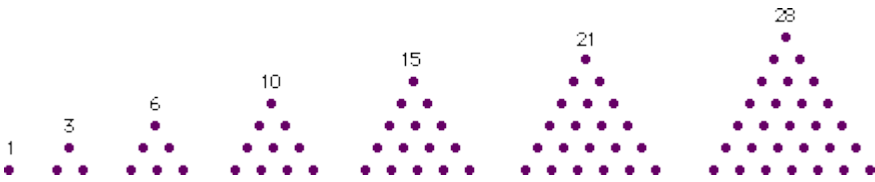


2. Fold it back open from the other side to expose a new face.

Keep flexing it and marking new sides as you find them. How many sides does a tetraflexagon have?

## Solutions

Monday

A.	19, 23, 27, ...	Each number is four greater than the previous number
B.	108, 324, 972, ...	Each number is the previous number multiplied by three
C.	$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$	Each number is the previous number divided by two
D.	15, 21, 28, ...	<p>These are the triangular numbers. Imagine build a triangle of dots. With each number you are adding a new row to the bottom of the triangle that is one longer than the previous bottom row.</p> 
E.	25, 36, 49, ...	These are the square numbers. We get these by multiplying each whole number by itself. ( $1 \times 1$ , $2 \times 2$ , $3 \times 3$ , $4 \times 4$ , $5 \times 5$ , etc)
F.	13, 21, 34, ...	These are the Fibonacci numbers. The sequence starts with two ones. After that each number is the sum of the previous two numbers.

Wednesday

- |            |           |              |             |           |
|------------|-----------|--------------|-------------|-----------|
| 1. XXXVIII | 2. LXXXIX | 3. CCCXXXIII | 4. MMCXLIII | 5. CDXCIV |
| 6. 28      | 7. 187    | 8. 436       | 9. 2097     | 10. 999   |

Friday

A tetraflexagon has six total sides. Twice as many as a hexaflexagon.

## **Magic Square**

Can you place the integers 1-9 in the boxes so that the sum of each row, column and diagonal is the same?
