Week 2 – Probability of a Compound Event (4/27/20 –5/1/20)

Welcome to distance learning. Assignments are required from now until then end of the school year. You will be graded on submitted material.

Goal: To continue our investigation of probability. **First**, we will use ordered lists, tables, and tree diagrams in order to determine the sample space of compound events. We will determine the probability of a compound event based on the event's sample space. **Second**, we will determine the theoretical probability of a compound event by calculation. This will involve determining the probability of each trial of a compound event individually and then combing these individual probabilities.

Contact		
Office hours by Email:	Mon – Fri: 8:00 AM – 3:30 PM mdibley@tusd.net	
Office hours by video:	Mon – Fri: 10:00 – 11:00 AM https://zoom.us/j/312003066 Mon – Fri: 3:00 – 4:00 PM https://zoom.us/j/218432703	Meeting ID: 312 003 066 Password: 805373 Meeting ID: 218 432 703 Password: 672048

How to get/return an assignment:	
Digital Option	non-Digital Option
• All digits lessons can be accessed through your digits account. • Videos, Notes, Content Practice (homework), etc. will all be uploaded to digits on (or before) Monday, April 27. • All practice problems from the digits lessons may be completed on paper or in a notebook. If you would like to use your actual Math Companion, it may be picked up from the George Kelly office on Friday, April 24. (Next distribution date: Friday May 8) • Digital assignments are submitted in the normal way. • Worksheets may be photographed and emailed or uploaded to digits.	• Lessons will be provided in a paper format. • A packet must be picked up from the George Kelly office on Friday, April 24. (Next distribution date: Friday May 8) • Your Math Companion may be picked up from the George Kelly Office on Friday, April 24. (Next distribution date: Friday May 8) • Completed assignments must be returned to the George Kelly office on Friday, May 8.

Digital Option:

1. Finding Theoretical Probability of a Compound Event by Determining the Sample Space
    a. Lesson 18.4: Finding Theoretical Probabilities (view the lessons and answer the Got It? Problems)
        i. Content Practice: 18-4 Homework G
        ii. Content Practice: Compound Events Worksheet
2. Finding Theoretical Probability of a Compound Event by Calculation
    a. Video: "Kolu Math – Compound Probability" (https://youtu.be/xLK0MWRwFYc)
        i. Notes: Theoretical Probability of a Compound Event
        i. Content Practice: Compound Events Worksheet #2

3. Bonus Logic Problem: Cake Cutting
    a. If you think you know the answer, send it to me in an email. Be sure to be clear in your explanation.

Digits 18.4 – Finding Theoretical Probabilities

Key Concept

From the tree diagram we can see that the possible outcomes are:

(R,R), (R,B), (R,G), (B,R), (B,B), (B,G), (G,R), (G, B), (G,G)

Of these, 3 have the same color twice.

Part 1

There are 5 cards: fish, start, apple, key and leaf. You are going to pick a card, record which one, replace it into the deck and then pick another card. The table shows all 25 possible results. Note that picking the fish and then the star is *different* then picking the star and then the fish.

To calculate the probabilities for each event, count up how many entries fit the event description and then divide by the total number of possible outcomes. (see next page)

Key Concept

When all outcomes of an action are equally likely, you can use the theoretical probability formula to find the probability of an event.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$P(\text{same color twice}) = \frac{3}{9}$$

3 favorable outcomes

9 possible outcomes

$$= \frac{1}{3}$$

Action
Spin twice.

Event
Spin the same color twice.

Sample space

Each section of the tree has one favorable outcome.

Part 1

Example **Finding Theoretical Probabilities of Compound Events**

The table shows the possible outcomes of choosing one card from the group shown, replacing it, and then choosing a second card. Find each probability.

a. *P*(at least one apple)
b. *P*(exactly one apple)
c. *P*(exactly one apple and one fish)
d. *P*(at least one apple or fish)

	F	S	A	K	L
F	F, F	S, F	A, F	K, F	L, F
S	F, S	S, S	A, S	K, S	L, S
A	F, A	S, A	A, A	K, A	L, A
K	F, K	S, K	A, K	K, K	L, K
L	F, L	S, L	A, L	K, L	L, L

Solution

Because you replace the card after the first choice, there are 5 ways to make each choice. So, there are 5 · 5, or 25, possible outcomes, as shown in the table.

continued on next page >

Part 1

Solution continued

a. The outcomes in the A column and A row are the outcomes that include apples. There are nine outcomes in the event *choose at least one apple.*

$P(\text{at least one apple}) = \frac{9}{25}$, or 36%

	F	S	A	K	L
F	F, F	S, F	A, F	K, F	L, F
S	F, S	S, S	A, S	K, S	L, S
A	F, A	S, A	A, A	K, A	L, A
K	F, K	S, K	A, K	K, K	L, K
L	F, L	S, L	A, L	K, L	L, L

b. The outcomes in the A column and A row are the outcomes that includes apples. Because you want outcomes that have exactly one apple, do not count the outcome (A, A). There are eight outcomes in the event *choose exactly one apple.*

$P(\text{exactly one apple}) = \frac{8}{25}$, or 32%

	F	S	A	K	L
F	F, F	S, F	A, F	K, F	L, F
S	F, S	S, S	A, S	K, S	L, S
A	F, A	S, A	A, A	K, A	L, A
K	F, K	S, K	A, K	K, K	L, K
L	F, L	S, L	A, L	K, L	L, L

c. Outcomes in the event *choose one apple and one fish* must include both A and F. There are two outcomes in this event.

$P(\text{one apple and one fish}) = \frac{2}{25}$, or 8%

	F	S	A	K	L
F	F, F	S, F	A, F	K, F	L, F
S	F, S	S, S	A, S	K, S	L, S
A	F, A	S, A	A, A	K, A	L, A
K	F, K	S, K	A, K	K, K	L, K
L	F, L	S, L	A, L	K, L	L, L

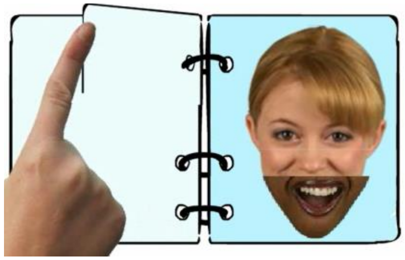
d. The outcomes in the F column, F row, A column, and A row are the outcomes that include at least one fish or one apple. There are 16 outcomes in this event.

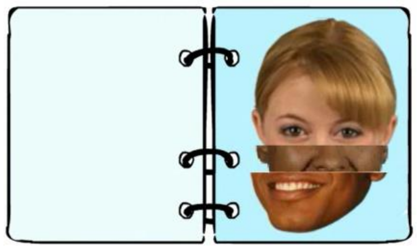
$P(\text{at least one apple or fish}) = \frac{16}{25}$, or 64%

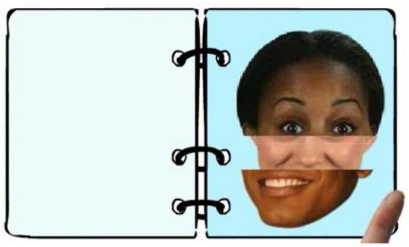
	F	S	A	K	L
F	F, F	S, F	A, F	K, F	L, F
S	F, S	S, S	A, S	K, S	L, S
A	F, A	S, A	A, A	K, A	L, A
K	F, K	S, K	A, K	K, K	L, K
L	F, L	S, L	A, L	K, L	L, L

Part 2

The flip book has three pictures of three of the hosts in it: Jay, Sara and Dana. The pages are split into three pieces. See examples below.







Why can we not use a table?

The tree diagram is used in order to put our choices into an organized list. There are 27 different combinations (remember the counting principle?). Of these, 18 of them have exactly two parts from the same person.

Part 2

Example Counting Outcomes to Find Theoretical Probabilities

In a certain book, you can arrange the eyes, noses, and mouths of the three people in various combinations. The people are Dana, Jay, and Sara.

You choose a combination at random. Use an organized list, a table, or a tree diagram to find the probability that your combination shows exactly two parts of the same person.

Solution

Sample: Use a tree diagram and find all of the outcomes that include exactly two parts of the same person.

Eyes	Nose	Mouth	Outcome	
Dana	Dana	Dana	Dana, Dana, Dana	
		Jay	Dana, Dana, Jay	✓
		Sara	Dana, Dana, Sara	✓
	Jay	Dana	Dana, Jay, Dana	✓
		Jay	Dana, Jay, Jay	✓
		Sara	Dana, Jay, Sara	
	Sara	Dana	Dana, Sara, Dana	✓
		Jay	Dana, Sara, Jay	
		Sara	Dana, Sara, Sara	✓
Jay	Dana	Dana	Jay, Dana, Dana	✓
		Jay	Jay, Dana, Jay	✓
		Sara	Jay, Dana, Sara	
	Jay	Dana	Jay, Jay, Dana	✓
		Jay	Jay, Jay, Jay	
		Sara	Jay, Jay, Sara	✓
	Sara	Dana	Jay, Sara, Dana	
		Jay	Jay, Sara, Jay	✓
		Sara	Jay, Sara, Sara	✓
Sara	Dana	Dana	Sara, Dana, Dana	✓
		Jay	Sara, Dana, Jay	
		Sara	Sara, Dana, Sara	✓
	Jay	Dana	Sara, Jay, Dana	
		Jay	Sara, Jay, Jay	✓
		Sara	Sara, Jay, Sara	✓
	Sara	Dana	Sara, Sara, Dana	✓
		Jay	Sara, Sara, Jay	✓
		Sara	Sara, Sara, Sara	

The sample space shows 27 outcomes, of which 18 include exactly two parts of the same person. The probability is $\frac{18}{27}$, or $66\frac{2}{3}\%$.

Part 3

We are assuming that we tried this experiment 330 times. The table shows how many times each of the six possible results occurred.

1. Calculate the experimental probability.
2. Determine how many times each event was expected to occur.
3. What is the theoretical probability of each event?

Part 3

Intro

You can run trials of a two-step action and compare experimental probabilities to theoretical probabilities.

To find an experimental probability, use this formula.

$$P(\text{event}) = \frac{\text{number of times event occurs}}{\text{total number of trials}}$$

Example Comparing Experimental and Theoretical Probabilities

A class ran 330 trials of this two-step action.

Action: Toss once. Spin once.

Complete the table. For which compound event is the count closest to the expected count?

Event	Count	Experimental Probability	Expected Count	Theoretical Probability
(H, R)	45			
(H, B)	59			
(H, G)	64			
(T, R)	45			
(T, B)	58			
(T, G)	59			

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Part 3 - Solution

$P(\text{event}) = \frac{\text{count}}{330}$

$330 \times \frac{1}{6} = 55$

$P(\text{event}) = \frac{1}{6}$

Event	Count	Experimental Probability	Expected Count	Theoretical Probability
(H, R)	45	0.1364	55	0.1667
(H, B)	59	0.1788	55	0.1667
(H, G)	64	0.1939	55	0.1667
(T, R)	45	0.1364	55	0.1667
(T, B)	58	0.1758	55	0.1667
(T, G)	59	0.1788	55	0.1667

The count for (toss tails, spin blue) is closest to its expected count.

Name ______________________________ Class ____________ Date ____________

Practice 17-4

Finding Theoretical Probabilities

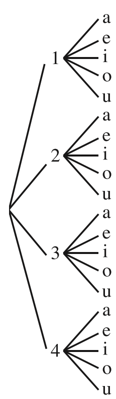
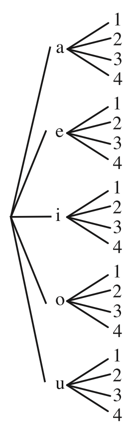
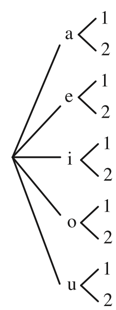
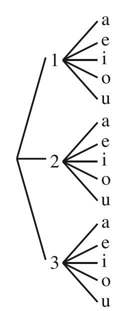
1. A fair coin is tossed two times in succession. The sample space is shown, where H represents a head and T represents a tail. Find the probability of getting exactly one tail.

Sample Space	
(Toss 1, Toss 2)	
(H,H)	(T,H)
(H,T)	(T,T)

2. The sample space shows the possible outcomes of picking two cards out of five different colored cards, red (R), green (G), blue (B), yellow (Y), and purple (P). Find the probability of picking one blue card and one green card.

Sample Space			
(R, G)	(R, B)	(R, Y)	(R, P)
(G, R)	(G, B)	(G, Y)	(G, P)
(B, R)	(B, G)	(B, Y)	(B, P)
(Y, R)	(Y, B)	(Y, G)	(Y, P)
(P, R)	(P, B)	(P, G)	(P, Y)

3. a) Select a tree diagram for choosing a vowel (a, e, i, o, u) and then a number (1, 2, 3, or 4).

❍ A.

❍ B.

❍ C.

❍ D.


b) Use the diagram to find the probability of choosing u and 2.

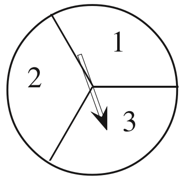
4. Three fair coins are tossed with possible outcomes of heads, H, and tails, T.

a) Complete the table.

b) Find the probability of tossing at least 2 heads.

Sample Space		
Toss 1	**Toss 2**	**Toss 3**
H	H	H
H	H	T
H	T	T
T	T	T
T	T	H
T	H	H
H	T	________
T	H	________

5. The table shows the result of spinning the wheel once and tossing a coin 44 times.



Results of 44 Trials	
Event	**Count**
(1, H)	6
(1, T)	10
(2, H)	6
(2, T)	7
(3, H)	8
(3, T)	7

a) Find the theoretical probability of spinning a three and tossing a tail. Simplify your answer.

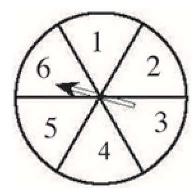
b) Find the experimental probability of spinning a three and tossing a tail. Simplify your answer.

6. A coin is tossed two times in succession. The results of 100 trials are shown in the table where H represents a coin landing on heads and T represents a coin landing on tails. For which compound event is the result closest to the result predicted by theoretical probability?

Results of 100 Trials	
Event	**Count**
HH	16
HT	23
TH	42
TT	19

❍ A. HT
❍ B. HH
❍ C. TT
❍ D. TH

7. **Writing** The table shows the possible outcomes of spinning the given spinner, where each section is equally likely, and tossing a fair coin.



a) Complete the table.

Sample Space						
	1	**2**	**3**	**4**	**5**	**6**
H	1,H	2,H	________	4,H	5,H	________
T	________	2,T	3,T	4,T	________	6,T

b) Find the probability of spinning a number greater than 1 or tossing a head.

c) How would the probability change if the spinner had eight equal parts instead of six?

8. **Reasoning** The table shows the results of rolling a number cube and tossing a coin 85 times.

Results of 85 Trials					
Event	**Count**	**Event**	**Count**	**Event**	**Count**
(1, H)	6	(2, H)	10	(3, H)	11
(1, T)	9	(2, 5)	5	(3, 5)	4
Event	**Count**	**Event**	**Count**	**Event**	**Count**
(4, H)	5	(5, H)	8	(6, H)	4
(4, T)	7	(5, T)	6	(6, T)	10

a) Find the theoretical probability of getting a tail.

b) Find the experimental probability of getting a tail.

c) Why would you want to use experimental instead of theoretical probability in some situations? Explain.

9. **Error Analysis** The sample space of picking a 2 character password using the letters Y, B, R, O, G, P, where double letters are not allowed is shown. Your teacher asks the class to find the probability of choosing a password with no Y's. Manuela incorrectly says that the probability is $\frac{3}{2}$.

Possible Combinations					
(Y, B)	(B, R)	(R, O)	(O, G)	(G, P)	(P, Y)
(Y, R)	(B, O)	(R, G)	(O, P)	(G, Y)	(P, B)
(Y, O)	(B, G)	(R, P)	(O, Y)	(G, B)	(P, R)
(Y, G)	(B, P)	(R, Y)	(O, B)	(G, R)	(P, O)
(Y, P)	(B, Y)	(R, B)	(O, R)	(G, O)	(P, G)

a) Find the correct probability. Simplify your answer.

b) Which error might have Manuela made?

❍ A. Manuela multiplied the number of possible outcomes by the number of favorable outcomes. She should have divided the number of favorable outcomes by the number of possible outcomes.

❍ B. Manuela divided the number of possible outcomes by the number of favorable outcomes. She should have divided the number of favorable outcomes by the number of possible outcomes.

10. **Geography** At this year's Geography Fair you and a partner are giving a presentation on one state in the Unites States. You need to pick a partner and a state for the presentation.

U.S. Geography Fair	
Student	**State**
Jackie	Hawaii, HI
Emily	Montana, MT
Katie	Delaware, DE
Mark	California, CA
Jimmy	

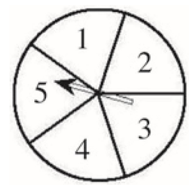
a) Complete the table for Jackie and Katie.

U.S. Geography Fair Sample Space					
Possibilities with Jackie		**Possibilities with Emily**		**Possibilities with Katie**	
Jackie	MT	Emily	MT	Katie	MT
Jackie	________	Emily	DE	Katie	DE
Jackie	CA	Emily	CA	Katie	CA
Jackie	HI	Emily	HI	Katie	________

Possibilities with Mark		**Possibilities with Jimmy**	
Mark	MT	Jimmy	MT
Mark	DE	Jimmy	DE
Mark	CA	Jimmy	CA
Mark	HI	Jimmy	HI

b) Use the table to find P(not Jackie and MT). Simplify your answer.

11. Mental Math The table shows the possible outcomes of spinning the given spinner and tossing a fair coin. Find the probability of spinning a 1, 2, or 4, and tossing a head.

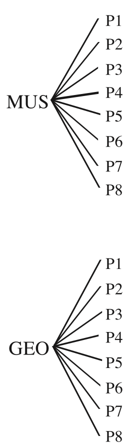


	1	2	3	4	5
H	1, H	2, H	3, H	4, H	5, H
T	1, T	2, T	3, T	4, T	5, T

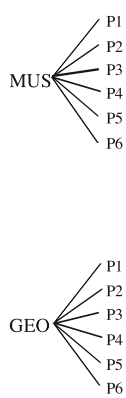
12. Zoe needs to take Music (MUS) or Geometry (GEO) this year. She can take the class during any one of eight periods (P1 through P8). At random, Zoe chooses a period and then a class.

a) Which tree diagram displays the sample space?

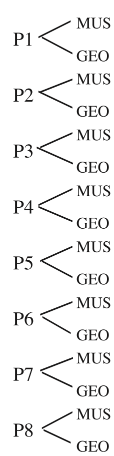
❍ A.



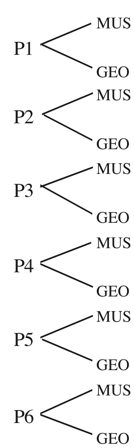
❍ B.



❍ C.



❍ D.



b) Find the probability that she chooses Geometry during period 2.

13. A pair of number cubes is rolled 169 times. The table shows the results of the sums.

Results of the Sums Formed by Rolling a Pair of Number Cubes 169 Times										
Event	2	3	4	5	6	7	8	10	11	12
Count	5	9	17	23	24	31	22	17	15	6

a) Find the theoretical probability of getting two numbers whose sum is 4. Simplify your answer.

b) Find the experimental probability of getting two numbers whose sum is 4. Simplify your answer.

14. Challenge You ask the salesperson at a pizza place to give you a pizza with any two toppings. The choices are shown in the table.

Pizza Toppings	
Mushrooms (M)	Broccoli (B)
Pepperoni (P)	Spinach (S)
Olives (O)	Tomatoes (T)

a) Complete the table to show the sample space.

Two Topping Pizzas					
(T,B)	(B,M)	(M,P)	(P,O)	(O,S)	(S,T)
(T,M)	(B,P)	(M,O)	(P,S)	(O,T)	(S,B)
(________,P)	(B,O)	(M,S)	(P,T)	(O,B)	(S,M)
(T,O)	(B,S)	(M,T)	(P,B)	(O,________)	(S,P)
(T,S)	(________,T)	(M,B)	(P,M)	(O,P)	(S,O)

b) Find the probability of getting a pizza without pepperoni.

15. Challenge A pair of number cubes are rolled. The numbers that are rolled are then multiplied together. This action is repeated 500 times. The table shows the results for some of the products. Using the products shown, which of the following events is the result closest to the result predicted by theoretical probability?

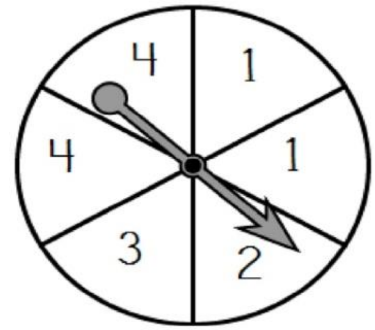
Results of 500 Trials	
Event	**Count**
4	30
6	77
8	14
10	23
12	72

❍ A. P(all products divisible by 2 and 4)

❍ B. P(all products divisible by 2 and 3)

❍ C. P(all products divisible by 2)

Refer to the spinner at right to answer the following questions. Your answers to questions #4 – #11 should be written as both a simplified fraction and a percent rounded to the nearest whole number. Circle both answers.



You spin the spinner twice.

1. What is the formula for theoretical probability?

2. How many outcomes are in the sample space for the two spins? Explain.

3. Create a table to show all possible outcomes from the two spins. (your table should have six rows and six columns)

For problems #4 – 11: Use the table you created in problem #3 to find how many outcomes are in the event.

4. What is the probability that you spin a 1 and then a 2?

5. What is the probability of spinning a 1 and then a 4?

6. What is the probability of spinning two 2's?

7. What is the probability of spinning at least one 3?

8. What is the probability of not spinning a 3 on either spin?

9. What is the probability of spinning two odd numbers?

10. What is the probability the sum of the two spins is even?

11. What is the probability that the sum of the two spins is a prime number?

12. If you spun the spinner three times, then how many outcomes are in the sample space? Explain.

Unit 4 –Probability

digits 18-4: "Theoretical Probability of a Compound Event"

Unit 4 4/27/20

Theoretical Probability of a Compound Event
(by Calculation)

- Begin on a new page
- Write the date and unit in the top corners of the page
- Write the title across the top line

Probability of a Compound Event

When two events occur in sequence (one after the other), the probability of the compound event is equal to the product of the probabilities for the individual events.

$$P(A\ and\ B) = P(A) \times P(B)$$

Example 1:

Two number cubes are rolled. What is the probability that both rolls are a six?

$$P(both\ are\ 6) = P(6) \times P(6) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Example 2:

You flip a coin 5 times in a row. What is the probability that all flips were heads?

$$P(all\ are\ heads) = P(H) \times P(H) \times P(H) \times P(H) \times P(H)$$

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}$$

Example 3:

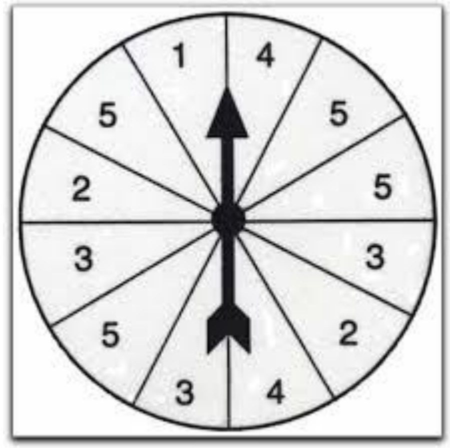
You pick two marbles from the following bag. What is the probability that both are green?

$$P(green, first\ pick) = \frac{4}{10}$$

One fewer green marble, because we assume our first pick was successful and we did not replace the marble

$$P(green, second\ pick) = \frac{3}{9}$$

$$P(both\ green) = \frac{4}{10} \times \frac{3}{9} = \frac{12}{90} = \frac{2}{15}$$



Refer to the spinner at right to answer the following questions. Show your work or you will lose points! Express all answers as a simplified fraction and as a percent rounded to the nearest tenth of a percent. Do NOT create a table, but calculate the probabilities based on the individual probabilities of the two events.

Remember: $P(Event\ A\ and\ then\ Event\ B) = P(A) \times P(B)$

Example: You are spinning the spinner two times. What is the probability of spinning a 2 both times?

$$P(2\ and\ then\ a\ 2) = P(2) \times P(2) = \frac{2}{12} \times \frac{2}{12} = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

1. You are spinning the spinner two times. How many outcomes are in the sample space?

2. You are spinning the spinner two times. What is the probability of spinning a 4 and then a 5?

3. You are spinning the spinner two times. What is the probability of spinning a 1 and then a 5?

4. You are spinning the spinner two times. What is the probability of spinning a 5 and then a 1?

5. You are spinning the spinner two times. What is the probability of spinning Two 1's?

6. You are spinning the spinner two times. What is the probability of spinning two even numbers?

7. You are spinning the spinner two times. What is the probability of not spinning a 3 on either spin?

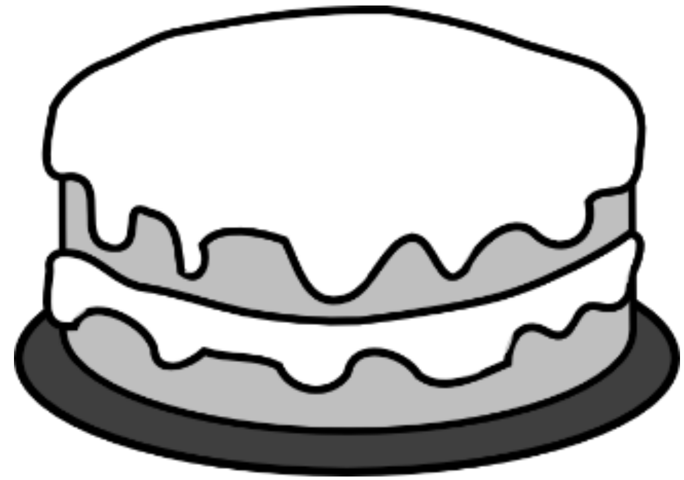
8. You are spinning the spinner two times. What is the probability than you spin a number greater than three on both spins?

9. You are spinning the spinner two times. What is the probability of spinning a 5 and then a 6?

10. You are spinning the spinner two times. What is the probability of spinning two odd numbers?

11. You are spinning the spinner three times. What is the probability of spinning a 5 on all three spins?

HOW IS IT POSSIBLE TO CUT A TRADITIONAL CIRCULAR CAKE INTO 8 EQUAL SIZE PIECES, WITH



ONLY 3 STRAIGHT CUTS?