Week 4 – Transversals and Triangles (5/11/20 –5/15/20)

Welcome to distance learning. Assignments are required from now until then end of the school year. You will be graded on submitted material.

Office Hour Schedule	
Monday	Lesson 25.1
Tuesday	Lesson 25.2
Wednesday	Lesson 25.3
Thursday	Lesson 25.4
Friday	Open Question and Answer

Goal: To understand what a transversal is. To understand the angle relationships created by a transversal and to use these relationships to determine if two lines are parallel. To understand the different angle relationships that are found in a triangle.

Contact		
Office hours by Email:	Mon – Fri: 8:00 AM – 3:30 PM mdibley@tusd.net	
Office hours by video:	Mon – Fri: 10:30 – 11:00 AM https://zoom.us/j/312003066 Mon – Fri: 3:30 – 4:00 PM https://zoom.us/j/218432703	Meeting ID: 312 003 066 Password: 805373 Meeting ID: 218 432 703 Password: 672048

How to get/return an assignment:	
Digital Option	non-Digital Option
• All digits lessons can be accessed through your digits account. • Videos, Notes, Content Practice (homework), etc. will all be uploaded to digits on (or before) Monday, May 11. • Digital assignments are submitted in the normal way. • Worksheets may be photographed and emailed or uploaded to digits.	• Lessons will be provided in a paper format. • A packet must be picked up from the George Kelly office on Friday May 8. This is the last pick-up day current scheduled. • Completed assignments must be returned to the George Kelly office on Friday, May 15. This is the last drop-off day current scheduled.

Digital Option:

1. (*digits* Topic 25) Reasoning in Geometry
    a. Video: "Parallel Lines Cut by a Transversal - Finding Angle Measures" (https://youtu.be/3Ex7SpsA9MI)
    b. Lesson 25.1: "Angles, Lines and Transversals" (view the lessons and answer the Got It? Problems)
    c. Lesson 25.2: "Reasoning and Parallel Lines" (view the lessons and answer the Got It? Problems)
    d. Lesson 25.3: "Interior Angles of Triangles" (view the lessons and answer the Got It? Problems)
    e. Lesson 25.4: "Exterior Angles of Triangles" (view the lessons and answer the Got It? Problems)
    f. Notes: Transversal and Triangles

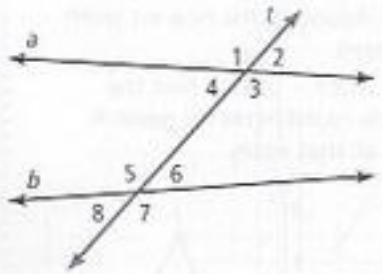
2. Content Practice
    a. 25-1 Homework G
    b. 25-2 Homework G
    c. 25-3 Homework G
    d. 25-4 Homework G
    e. Worksheet: "Angles on the Plains of Nazca"

3. Bonus Logic Problem: Checkerboard
    a. This one is quite challenging. I suggest you try a smaller problem, like a 3x3 checkerboard first. (or, maybe a 2x2. Or … how about a 1x1?)
    b. If you would like a clue, just ask.

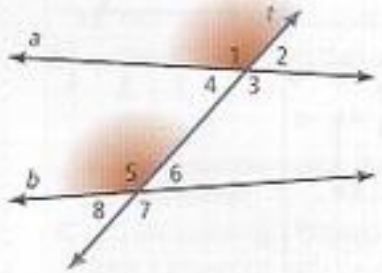
Digits 11-1: Angles, Lines and Transversals

Key Concept

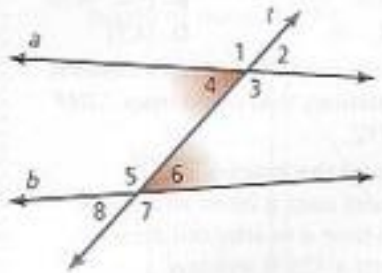
A **transversal** is a line that intersects two or more lines at different points. In the diagram, line *t* is a transversal.



Corresponding angles lie on the same side of a transversal and in corresponding positions. There are four pairs of corresponding angles in this diagram: ∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, and ∠4 and ∠8.



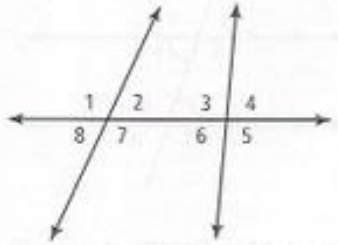
Alternate interior angles lie within a pair of lines and on opposite sides of a transversal. There are two pairs of alternate interior angles in this diagram: ∠3 and ∠5, and ∠4 and ∠6.



Part 1

Example **Identifying Corresponding Angles and Alternate Interior Angles**

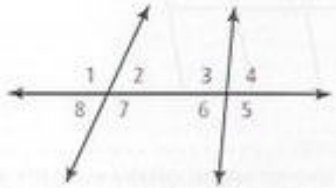
Name the pairs of corresponding angles and the pairs of alternate interior angles.



Solution

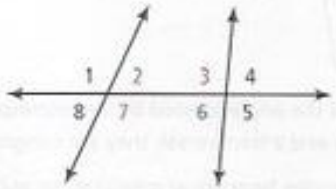
Corresponding angles lie on the same side of the transversal and in corresponding positions. The following pairs of angles are corresponding angles:

∠1 and ∠3
∠2 and ∠4
∠5 and ∠7
∠6 and ∠8



Alternate interior angles lie within a pair of lines and on opposite sides of the transversal. The following pairs of angles are alternate interior angles:

∠2 and ∠6
∠3 and ∠7

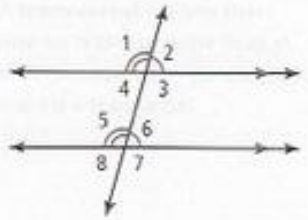


Digits 11-1: Angles, Lines and Transversals

Part 2

Intro

When a transversal intersects two parallel lines, corresponding angles are congruent. Congruent angles have equal measures. You can mark angles with arcs to show that they are congruent.

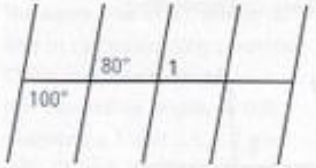


$\angle 1 \cong \angle 5$

$\angle 2 \cong \angle 6$

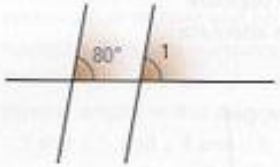
Example Finding Measures of Corresponding Angles

The segments that form the parking spaces are parallel. What is $m\angle 1$?



Solution

Identify the parallel lines and transversal need to find $m\angle 1$.



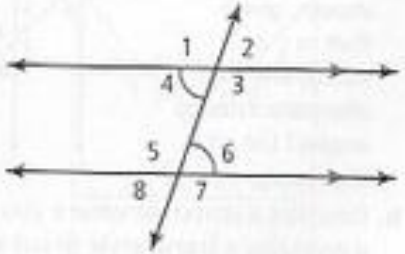
Since $\angle 1$ and the angle labeled 80° are corresponding angles formed by parallel lines and a transversal, they are congruent.

Congruent angles have equal measures, so $m\angle 1 = 80°$.

Part 3

Intro

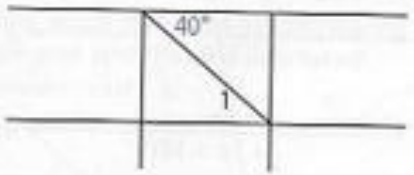
When a transversal intersects two parallel lines, alternate interior angles are congruent.



$\angle 4 \cong \angle 6$

Example Finding Measures of Alternate Interior Angles

Andie is working on a dude ranch. She is repairing fences. The rails of the fence shown are parallel. What is $m\angle 1$?



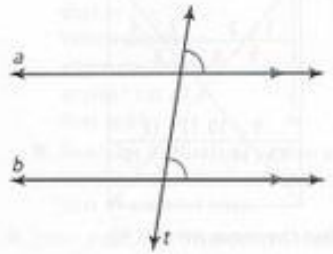
Solution

Since $\angle 1$ and the angle labeled 40° are alternate interior angles formed by parallel lines and a transversal, they are congruent. Congruent angles have equal measures, so $m\angle 1 = 40°$.

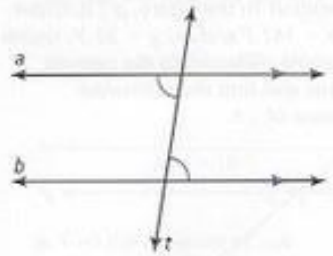
Digits 11-2: Reasoning and Parallel Lines

Key Concept

Corresponding Angles and Parallel Lines If the corresponding angles formed by two lines and a transversal are congruent, then the lines are parallel.



Alternate Interior Angles and Parallel Lines If the alternate interior angles formed by two lines and a transversal are congruent, then the lines are parallel.



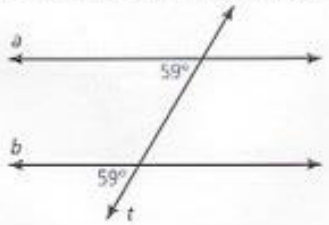
Part 1

Intro

The symbol ∥ means "is parallel to." If line m is parallel to line n, you write $m \parallel n$.

Example Justifying Parallel Lines with Corresponding Angles

Can you conclude that $a \parallel b$? Justify your reasoning.



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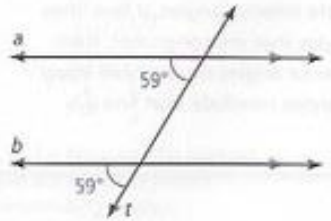
Part 1

Example continued

Solution



The angles labeled 59° are corresponding angles formed by two lines and a transversal. The angles have equal measures, so they are congruent. If two lines and a transversal form corresponding angles that are congruent, then the lines are parallel. So you can conclude that $a \parallel b$.

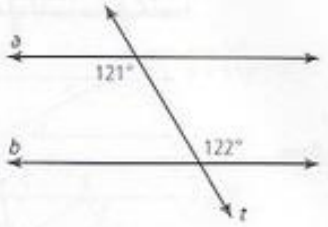
Part 2

Intro

The reasoning that you use to decide whether two lines are parallel based on knowing whether corresponding angles or alternate interior angles are congruent is called deductive reasoning. **Deductive reasoning** is a process of reasoning logically from given facts to a conclusion.

Example Justifying Parallel Lines with Alternate Interior Angles

Can you conclude that $a \parallel b$? Justify your reasoning.



Digits 11-2: Reasoning and Parallel Lines

Example continued

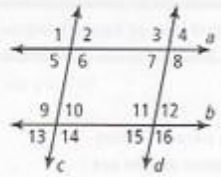
Solution

The angles labeled 121° and 122° are alternate interior angles. If two lines and a transversal form alternate interior angles that are congruent, then the lines are parallel. Since the alternate interior angles do *not* have equal measures, they are *not* congruent. So you cannot conclude that line *a* is parallel to line *b*.

Part 3

Example **Using Angle Congruence to Justify Parallel Lines**

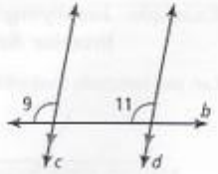
Which congruence statements justify $a \parallel b$ or $c \parallel d$?



$\angle 2 \cong \angle 10$
$\angle 3 \cong \angle 6$
$\angle 4 \cong \angle 12$
$\angle 7 \cong \angle 12$
$\angle 9 \cong \angle 11$
$\angle 10 \cong \angle 15$

Solution

If two lines and a transversal form corresponding angles that are congruent, then the lines are parallel.

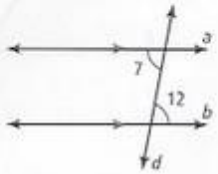


If $\angle 9 \cong \angle 11$, then $c \parallel d$.

If $\angle 2 \cong \angle 10$, then $a \parallel b$.

If $\angle 4 \cong \angle 12$, then $a \parallel b$.

If two lines and a transversal form alternate interior angles that are congruent, then the lines are parallel.



If $\angle 7 \cong \angle 12$, then $a \parallel b$.

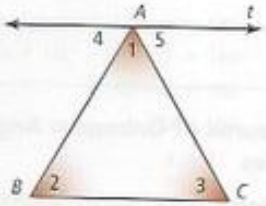
If $\angle 3 \cong \angle 6$, then $c \parallel d$.

If $\angle 10 \cong \angle 15$, then $c \parallel d$.

Digits 11-3: Interior Angles of Triangles

Key Concept

The sum of the measures of the interior angles of a triangle is 180°.



Line *t* is $\parallel$ to $\overline{BC}$, so alternate interior angles are —.

$m\angle 4 = m\angle 2$
$m\angle 5 = m\angle 3$

The measure of a straight angle is 180°.

$m\angle 4 + m\angle 1 + m\angle 5 = 180°$

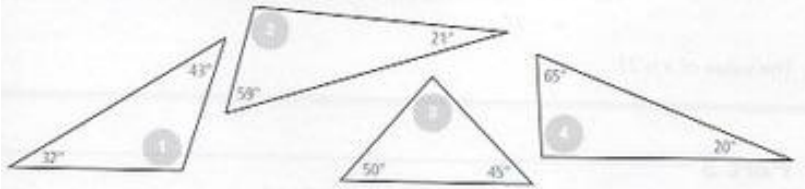
Substitution.

$m\angle 2 + m\angle 1 + m\angle 3 = 180°$

Part 1

Example **Finding Measures of Third Angles in Triangles**

Find the missing angle measure in each triangle.



Solution

For each triangle, write an equation for the sum of the measures of its interior angles. Then solve for the missing angle measure.

Measure of $\angle 1$

$m\angle 1 + 43° + 32° = 180°$
$m\angle 1 + 75° = 180°$
$m\angle 1 = 105°$

Measure of $\angle 2$

$m\angle 2 + 59° + 21° = 180°$
$m\angle 2 + 80° = 180°$
$m\angle 2 = 100°$

Digits 11-3: Interior Angles of Triangles

Solution continued

Measure of $\angle 3$

$$m\angle 3 + 50^\circ + 45^\circ = 180^\circ$$
$$m\angle 3 + 95^\circ = 180^\circ$$
$$m\angle 3 = 85^\circ$$

Measure of $\angle 4$

$$m\angle 4 + 20^\circ + 65^\circ = 180^\circ$$
$$m\angle 4 + 85^\circ = 180^\circ$$
$$m\angle 4 = 95^\circ$$

Part 2

Example **Finding Measures of Unknown Angles in Right Triangles**

The diagram shows a ladder against a wall and the angle that the ladder makes with the ground. What is the value of x?

Solution

The ladder forms a triangle with the side of the wall and the ground. Write and solve an equation for the sum of the measures of the interior angles of the triangle to find the value of x.

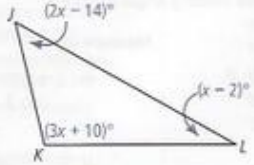
$$63^\circ + 90^\circ + x^\circ = 180^\circ$$
$$153 + x = 180$$
$$x = 27$$

The value of x is 27.

Part 3

Example **Finding Measures of Interior Angles of Triangles**

What are the measures of the angles of $\triangle JKL$?



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Part 3

Example continued

Solution

The sum of the measures of the interior angles of a triangle is 180°.

Step 1 Write an equation to find the value of x.

$$m\angle J + m\angle K + m\angle L = 180^\circ$$
$$(2x - 14)^\circ + (3x + 10)^\circ + (x - 2)^\circ = 180^\circ$$
$$6x - 6 = 180$$
$$6x = 186$$
$$x = 31$$

Step 2 Substitute the value of x into the expression for each angle measure.

$$m\angle J = (2x - 14)^\circ = [2(31) - 14]^\circ = (62 - 14)^\circ = 48^\circ$$
$$m\angle K = (3x + 10)^\circ = [3(31) + 10]^\circ = (93 + 10)^\circ = 103^\circ$$
$$m\angle L = (x - 2)^\circ = (31 - 2)^\circ = 29^\circ$$

The measures of the angles of $\triangle JKL$ are 48°, 103°, and 29°.

Check

$$m\angle J + m\angle K + m\angle L = 180^\circ$$
$$48^\circ + 103^\circ + 29^\circ \stackrel{?}{=} 180^\circ$$
$$180^\circ = 180^\circ \checkmark$$

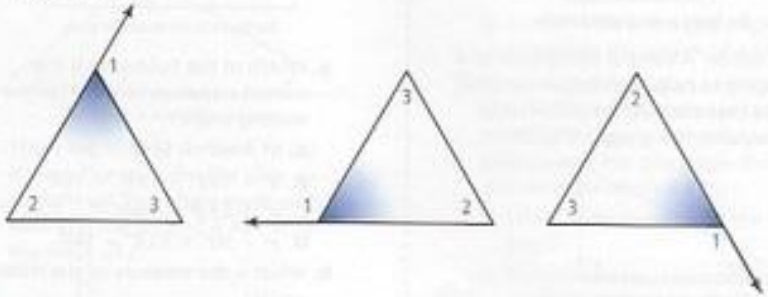
Digits 11-4: Exterior Angles of Triangles

Part 1

Intro

An **exterior angle of a triangle** is an angle formed by a side and an extension of an adjacent side. ∠1 is an exterior angle of each triangle.

For each exterior angle of a triangle, the two nonadjacent interior angles are its **remote interior angles**. ∠2 and ∠3 are remote interior angles of ∠1 in each triangle.

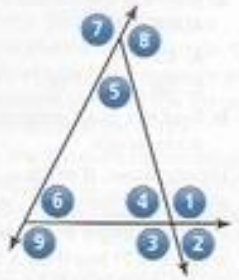


Example Identifying Interior and Exterior Angles of Triangles

a. Which of the numbered angles are exterior angles?

b. Name the remote interior angles for each exterior angle.

c. Which exterior angles are congruent? Explain.



Solution

a. Angles 1, 3, 8, and 9 are exterior angles of the triangle because they are angles formed by a side and an extension of an adjacent side.

b. Angles 5 and 6 are the remote interior angles for angle 1.
Angles 5 and 6 are the remote interior angles for angle 3.
Angles 4 and 5 are the remote interior angles for angle 9.
Angles 4 and 6 are the remote interior angles for angle 8.

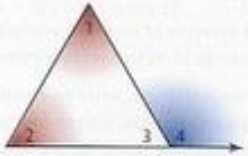
c. Angles 1 and 3 are vertical angles. Vertical angles have equal measures, so they are congruent.

Key Concept

The measure of an exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

Here is an example.

$m\angle 1 + m\angle 2 + m\angle 3 = 180°$ because the sum of the measures of the interior angles of a triangle equals 180°.



$m\angle 3 + m\angle 4 = 180°$ because ∠3 and ∠4 form a straight angle.

Substitute $m\angle 3 + m\angle 4$ for 180° in the first equation. Then subtract ∠3 from both sides.

$$m\angle 1 + m\angle 2 + m\angle 3 = m\angle 3 + m\angle 4$$
$$m\angle 1 + m\angle 2 + m\angle 3 - m\angle 3 = m\angle 3 + m\angle 4 - m\angle 3$$
$$m\angle 1 + m\angle 2 = m\angle 4$$

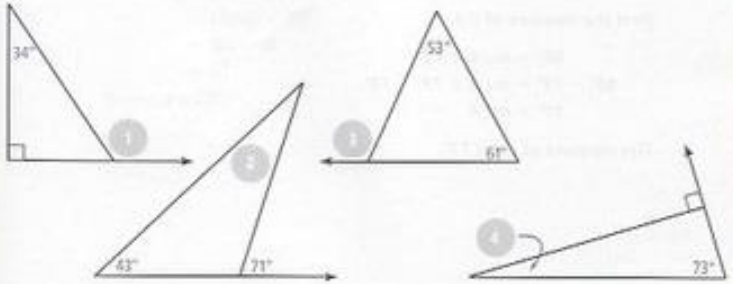
So the measure of an exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

Part 2

Example Finding Measures of Interior and Exterior Angles of Triangles

Find the missing angle measure in each diagram.

8° 28° 27° 17° 114° 163° 56° 124°



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Digits 11-4: Exterior Angles of Triangles

Example continued

Solution

The measure of an exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

For each triangle, write an equation that sets the measure of the exterior angle equal to the sum of the measures of its two remote interior angles. Then solve for the missing angle measure.

Find the measure of $\angle 1$.

$$\begin{aligned} m\angle 1 &= 34^\circ + 90^\circ \\ &= 124^\circ \end{aligned}$$

The measure of $\angle 1$ is 124°.

Find the measure of $\angle 2$.

$$\begin{aligned} 71^\circ &= m\angle 2 + 43^\circ \\ 71^\circ - 43^\circ &= m\angle 2 + 43^\circ - 43^\circ \\ 28^\circ &= m\angle 2 \end{aligned}$$

The measure of $\angle 2$ is 28°.

Find the measure of $\angle 3$.

$$\begin{aligned} m\angle 3 &= 53^\circ + 61^\circ \\ &= 114^\circ \end{aligned}$$

The measure of $\angle 3$ is 114°.

Find the measure of $\angle 4$.

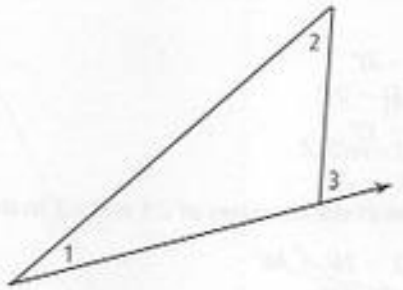
$$\begin{aligned} 90^\circ &= m\angle 4 + 73^\circ \\ 90^\circ - 73^\circ &= m\angle 4 + 73^\circ - 73^\circ \\ 17^\circ &= m\angle 4 \end{aligned}$$

The measure of $\angle 4$ is 17°.

Part 3

Example **Calculating Measures of Exterior Angles of Triangles**

Given $m\angle 1 = 26^\circ$, $m\angle 2 = (3x - 2)^\circ$, and $m\angle 3 = (5x - 8)^\circ$, what is $m\angle 3$?



Solution

The measure of an exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

Step 1 Write an equation to find the value of x.

$$\begin{aligned} m\angle 3 &= m\angle 1 + m\angle 2 \\ (5x - 8)^\circ &= 26^\circ + (3x - 2)^\circ \\ 5x - 8 &= 3x + 24 \\ 2x &= 32 \\ x &= 16 \end{aligned}$$

Step 2 Substitute the value of x into the expression for $m\angle 3$.

$$\begin{aligned} m\angle 3 &= (5x - 8)^\circ \\ &= [5(16) - 8]^\circ \\ &= (80 - 8)^\circ \\ &= 72^\circ \end{aligned}$$

So $m\angle 3$ is 72°.

Digits 11-4: Exterior Angles of Triangles

Solution continued

Check

Step 1 Find $m\angle 2$.

$$\begin{aligned} m\angle 2 &= (3x - 2)^\circ \\ &= [3(16) - 2]^\circ \\ &= (48 - 2)^\circ \\ &= 46^\circ \end{aligned}$$

Step 2 Compare the sum of the measures of $\angle 1$ and $\angle 2$ to the measure of $\angle 3$.

$$\begin{aligned} m\angle 1 + m\angle 2 &= 26^\circ + 46^\circ \\ &= 72^\circ \\ &= m\angle 3 \checkmark \end{aligned}$$

Unit 5 5/11/20

Transversals and Triangles

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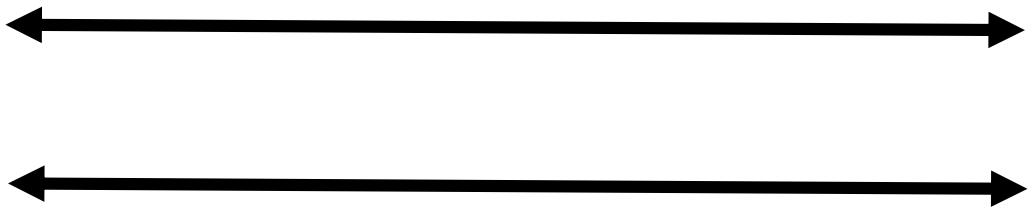
- Write the date and unit in the top corners of the page
- Write the title across the top line

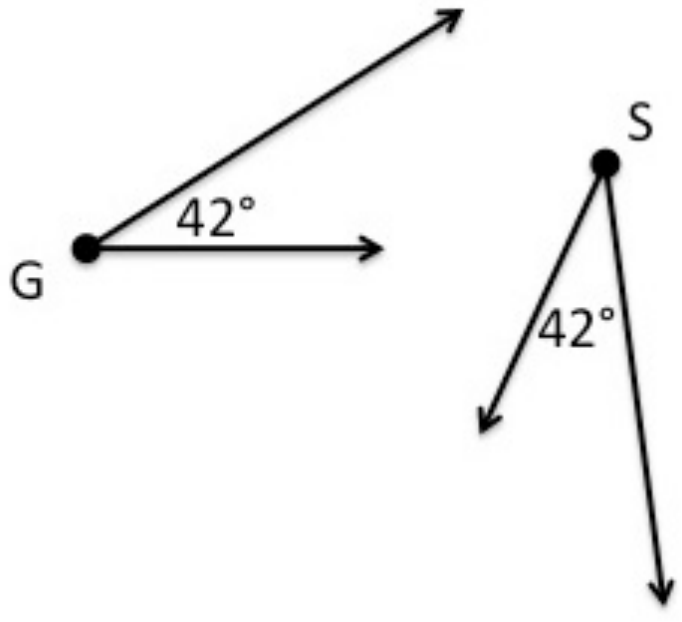
Parallel Lines

Two lines in the same plane that never intersect.



Congruent

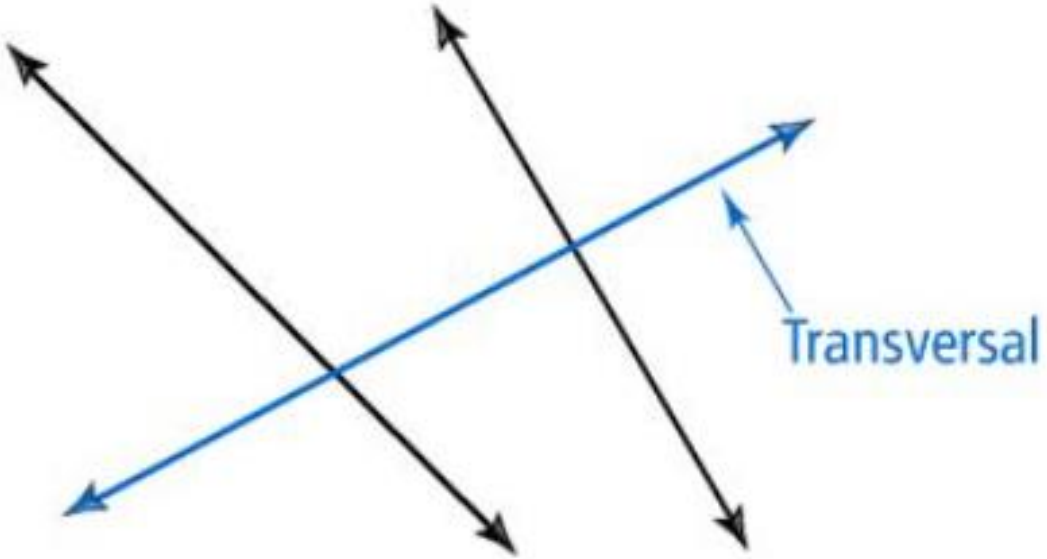
- Two figures that have the same shape and size.
- Two angles are congruent if they have the same measure.



$\angle G \text{ and } \angle S \text{ are congruent}$

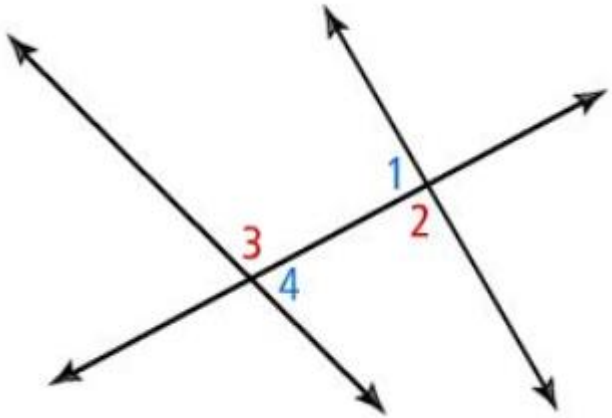
Transversal

A line that intersects two or more lines at different points.



Alternate Interior Angles

Alternate interior angles lie within a pair of lines and on opposite sides of a transversal.

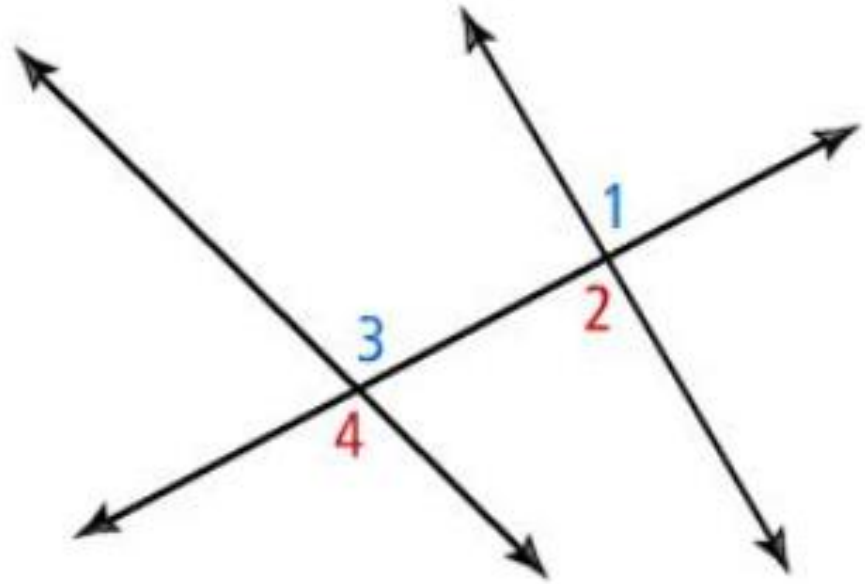


∠1 and ∠4 are alternate interior angles.

∠2 and ∠3 are also alternate interior angles.

Corresponding Angles

Angles that lie on the same side of a transversal and in corresponding positions.



∠1 and ∠3 are corresponding angles.

∠2 and ∠4 are also corresponding angles.

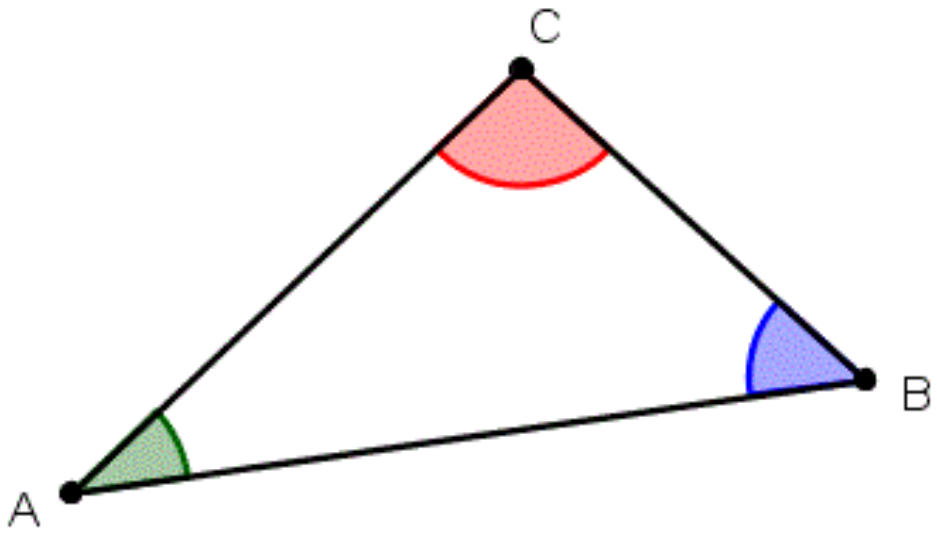
There are two more pairs of corresponding angles.

Deductive Reasoning

A process of reasoning logically from given facts to a conclusion.

Interior Angles of a Triangle

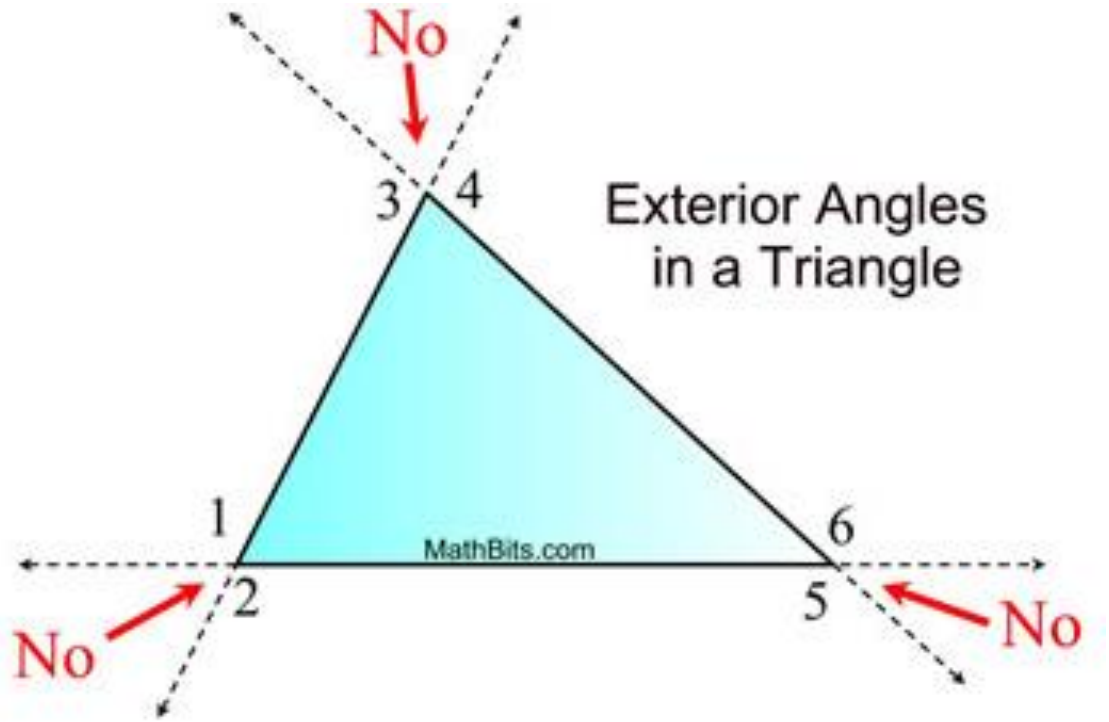
The sum of the interior angles of a triangle is 180°.



$$m\angle A + m\angle B + m\angle C = 180$$

Exterior Angle of a Triangle

An exterior angle of a triangle is an angle formed by a side and an extension of an adjacent side.

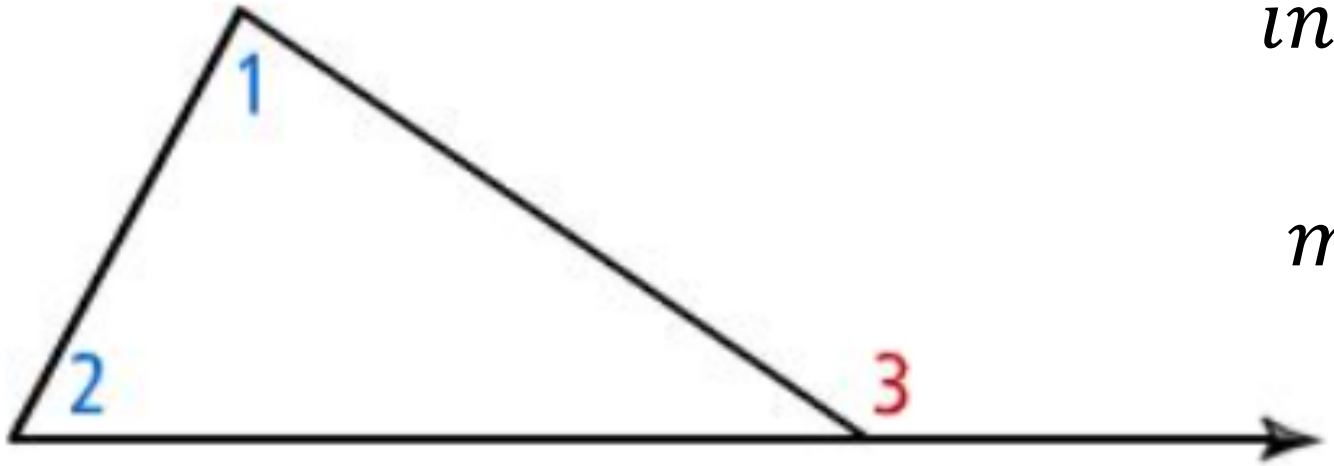


∠1, ∠2, ∠3, ∠4, ∠5, and ∠6 are all Exterior angles of this triangle.

Remote Interior Angles

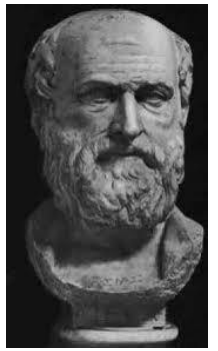
- The two nonadjacent interior angles corresponding to each exterior angle of a triangle.
- The measure of an exterior angle is equal to the sum of the two remote interior angles.

$\angle 1 \; and \angle 2 \; are \; remote \; interior \; angles \; to \; \angle 3$



$$m\angle 1 + m\angle 2 = m\angle 3$$

Eratosthenes (276 – 194 B.C.E)



Over 2200 years ago!

Measuring the Earth

Eratosthenes heard about a famous well in the Egyptian city of Swenet (Syene in Greek, and now known as Aswan), on the Nile River. At noon one day each year — the summer solstice (between June 20 and June 22) — the Sun's rays shone straight down into the deep pit. They illuminated only the water at the bottom, not the sides of the well as on other days, proving that the Sun was directly overhead. He reasoned that if the rays continued, they would pass through the center of the Earth.

Eratosthenes erected a pole in Alexandria (575 miles to the north), and on the summer solstice he observed that it cast a shadow, due to the fact that the Earth is round. (see diagram on next slide). Recognizing the curvature of the Earth and knowing the distance between the two cities enabled Eratosthenes to calculate the planet's circumference.

$m\angle 2 \approx \frac{1}{50}$ of a circle

He measured this value in Alexandria.

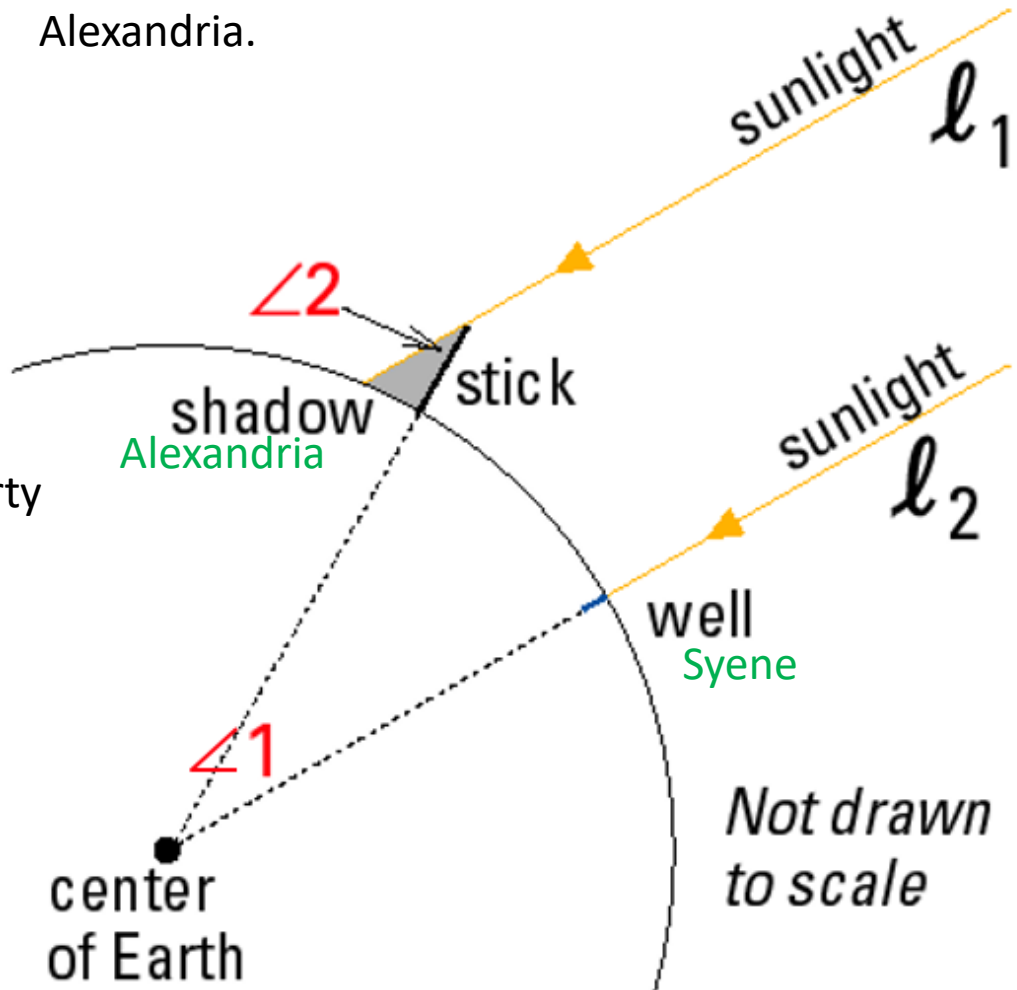
Using properties of parallel lines, he knew that

$m\angle 1 = m\angle 2$

Which property did he use?

He reasoned that

$m\angle 1 \approx \frac{1}{50}$ of a circle



He reasoned that the rays of sunlight in Alexandria were parallel to the rays of sunlight in Syene due to the size of the Sun and its distance from Earth.

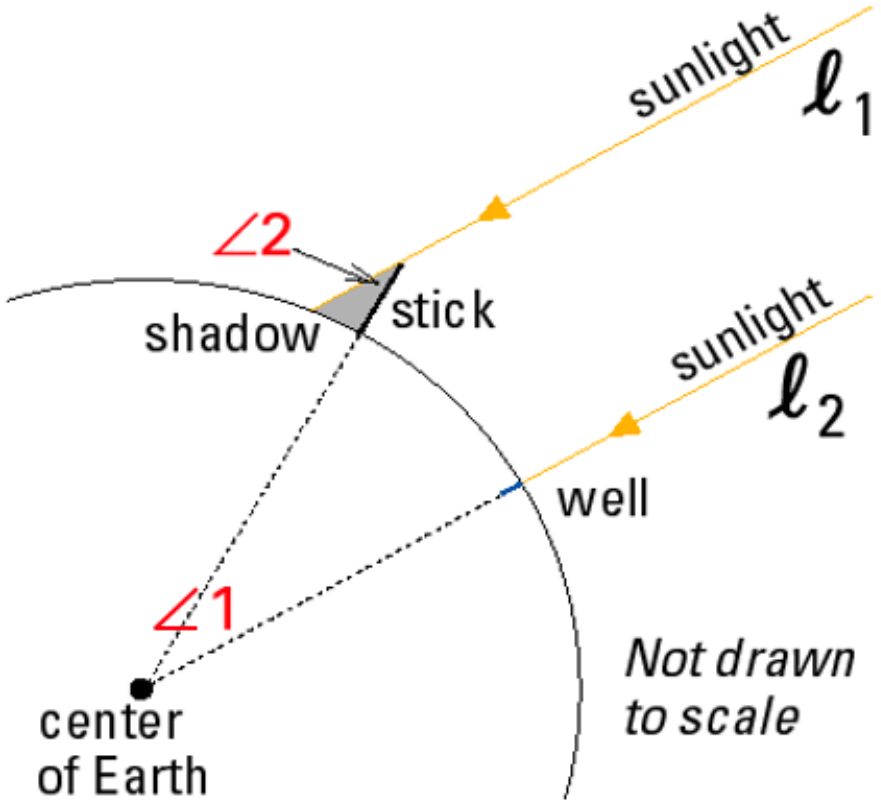
$m\angle 1 \approx \frac{1}{50}$ of a circle

The distance from Syene to Alexandria was believed to be **575 miles**

$$\frac{1}{50} \text{ of a circle} \approx \frac{\textbf{575 miles}}{\text{Earth's circumference}}$$

$$\text{Earth's circumference} \approx 50(575 \text{ miles})$$

$$\approx 29{,}000 \text{ miles}$$



Actual circumference = 24,900 miles

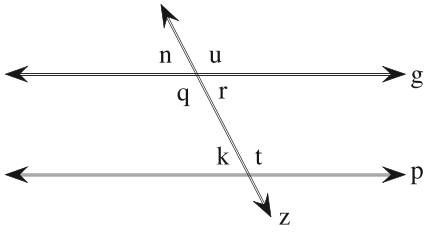
The biggest error was in his measurement was the distance between Alexandria and Syene.

Name _____ Class _____ Date _____

Practice 25-1

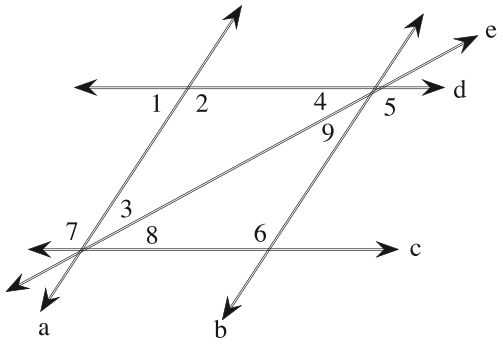
Angles, Lines, and Transversals

1. In the figure, g ∥ p. Which angles are alternate interior angles? Check all that apply.



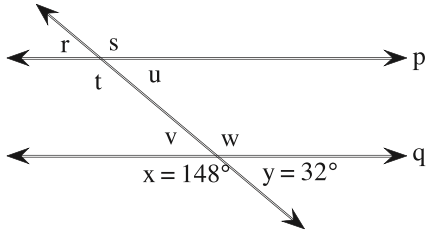
- ❑ A. ∠u and ∠n
- ❑ B. ∠q and ∠t
- ❑ C. ∠q and ∠n
- ❑ D. ∠r and ∠u
- ❑ E. ∠r and ∠k
- ❑ F. ∠u and ∠q

2. Which of the following is a pair of corresponding angles?

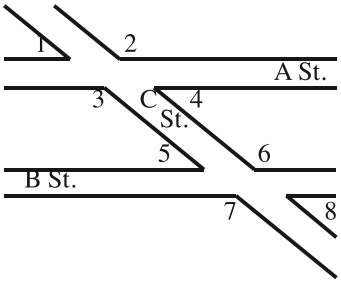


- ❍ A. ∠5 and ∠6
- ❍ B. ∠6 and ∠7
- ❍ C. ∠6 and ∠8
- ❍ D. ∠1 and ∠4

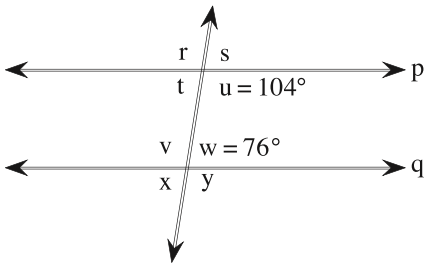
3. Find the measure of ∠u given that p ∥ q.



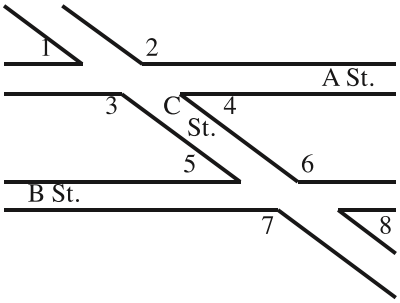
4. Streets A and B run parallel to each other. The measure of ∠8 is 23°. The measure of ∠6 is 157°. Find the measure of ∠2.



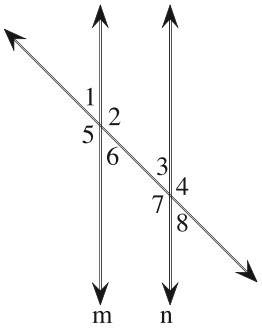
5. Find the measure of $\angle v$ given that p || q.



6. Streets A and B run parallel to each other. The measure of $\angle 6$ is 155°. The measure of $\angle 5$ is 25°. Find the measure of $\angle 4$.



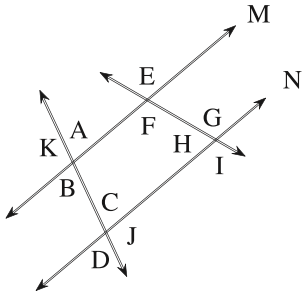
7. a) Writing Find the alternate interior angles in the figure shown, given that m || n. Which are the alternate interior angles? Check all that apply.



- ❑ A. $\angle 1$ and $\angle 7$
- ❑ B. $\angle 1$ and $\angle 8$
- ❑ C. $\angle 3$ and $\angle 6$
- ❑ D. $\angle 2$ and $\angle 5$
- ❑ E. $\angle 2$ and $\angle 7$
- ❑ F. $\angle 3$ and $\angle 8$

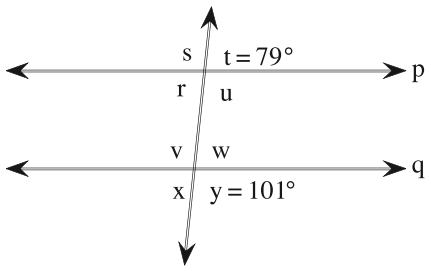
b) Describe a situation where you would use a transversal to cut a pair of parallel lines.

8. Reasoning Are $\angle K$ and $\angle B$ corresponding angles?



- ❍ A. No, because the angles do not have the same measure.
- ❍ B. No, because the angles do not lie on the same side of the transversal and in corresponding positions.
- ❍ C. Yes, because the angles lie on the same side of the transversal and in corresponding positions.
- ❍ D. Yes, all angles that have the same measure are corresponding angles.

9. Error Analysis On a recent math test a teacher asked for the measure of ∠w. In the figure, p ∥ q. Jacob incorrectly said that the measure was 101°.



a) Find the measure of ∠w.

b) Which error might Jacob have made?

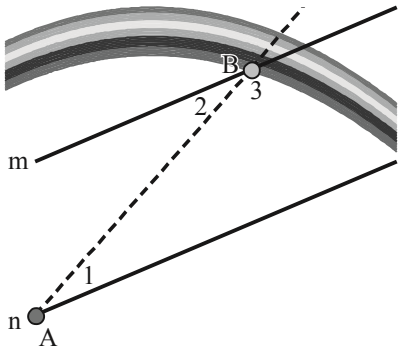
❍ A. Jacob thought that ∠w and ∠y are corresponding angles, when actually ∠w and ∠t are corresponding angles and the sum of their measures is 180°.

❍ B. Jacob thought that ∠w and ∠y are corresponding angles, when actually ∠w and ∠t are corresponding angles and have the same measure.

❍ C. Jacob thought that ∠w and ∠t are corresponding angles, when actually ∠w and ∠y are corresponding angles and have the same measure.

❍ D. Jacob thought that ∠w and ∠t are corresponding angles, when actually ∠w and ∠y are corresponding angles and the sum of their measures is 180°.

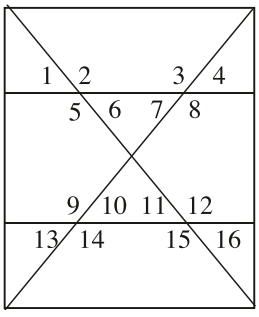
10. Rainbows When sunlight enters a drop of rain, different colors of light leave the drop at different angles, making a rainbow. In the figure shown, lines m and n represent the sun's rays. Assume that lines m and n are parallel and you are standing at point A.



a) For violet light at point B, m∠2 = 27° and m∠3 = 153°. What is m∠1?

b) Justify how you can determine m∠1.

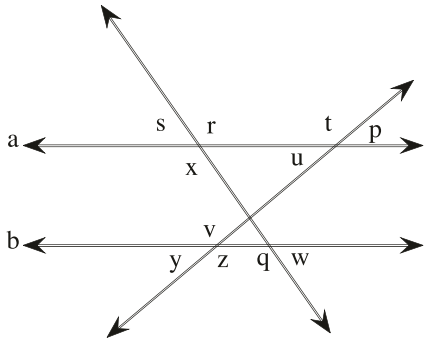
11. Open-Ended The figure shows the design of a rectangular window pane. Note that all horizontal lines are parallel. The measure of ∠6 is 53°. The measure of ∠2 is 127°.



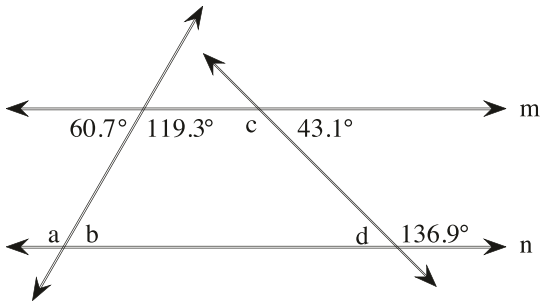
a) Find the measure of ∠12.

b) What other objects in your home have corresponding angles?

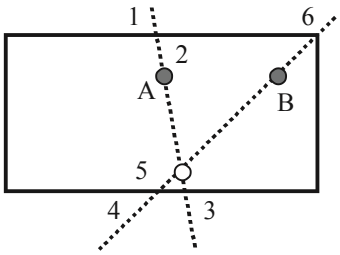
12. Estimation In the figure, a ∥ b. Given m∠x = 147.2° and m∠y = 32.8°, round the angle measures to the nearest degree and find the estimated measures of ∠u and ∠q.



13. Find the measures of ∠b and ∠d given that m ∥ n.



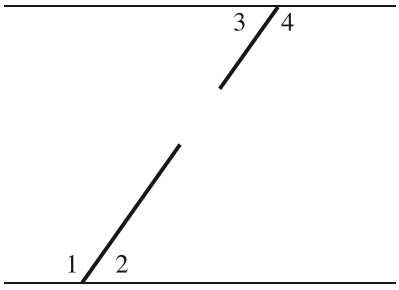
14. Challenge The figure shows two possible shots in a game of pool. The easiest shots to make in pool are shots where the corresponding angles are closest to 90°. The measure of ∠1 is 86°. The measure of ∠4 is 51°.



a) Find the measures of ∠3 and ∠6.

b) Should you aim for Ball A or Ball B?

15. Challenge Engineers are laying pipe below ground on opposite sides of the street as shown here. To join the pipe, workers on each side of the street work towards the middle. One team of workers lays the pipe using m∠4 = 117°. The other team of workers lays the pipe using m∠2. Find m∠2. Assume that the sides of the street are parallel and the pipe is straight.

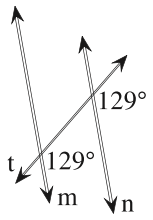


Name _____ Class _____ Date _____

Practice 25-2

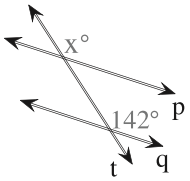
Reasoning and Parallel Lines

1. For the figure shown, decide if m || n.



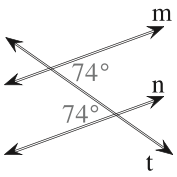
- ❍ A. Yes, m || n because the labeled angles are supplementary corresponding angles.
- ❍ B. No, line m is not parallel to line n because the labeled angles are corresponding angles, but they are not congruent.
- ❍ C. No, line m is not parallel to line n because the labeled angles are congruent, but they are not corresponding angles.
- ❍ D. Yes, m || n because the labeled angles are congruent corresponding angles.

2. In order for line p to be parallel to line q, what must be the value of x?



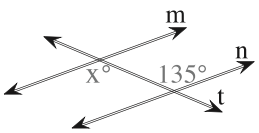
(The figure is not to scale.)

3. For the given figure, can you conclude m || n?

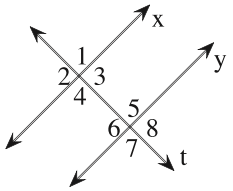


- ❍ A. No, line m is not parallel to line n because the labeled angles are congruent, but they are not alternate interior angles.
- ❍ B. No, line m is not parallel to line n because the labeled angles are alternate interior angles, but they are not congruent.
- ❍ C. Yes, m || n because the labeled angles are congruent alternate interior angles.
- ❍ D. Yes, m || n because the labeled angles are supplementary alternate interior angles.

4. What value of x will make line m parallel to line n?

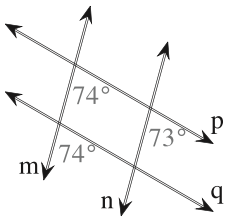


5. Find a congruence statement that justifies x ∥ y.



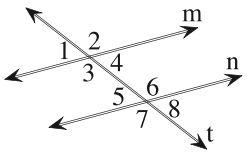
- ❍ A. If ∠5 ≅ ∠6, then x ∥ y because if alternate interior angles are congruent, then the lines are parallel.
- ❍ B. If ∠3 ≅ ∠6, then x ∥ y because if corresponding angles are congruent, then the lines are parallel.
- ❍ C. If ∠2 ≅ ∠6, then x ∥ y because if alternate interior angles are congruent, then the lines are parallel.
- ❍ D. If ∠1 ≅ ∠5, then x ∥ y because if corresponding angles are congruent, then the lines are parallel.

6. Which lines, if any, are parallel?



- ❍ A. m ∥ n and p ∥ q
- ❍ B. m ∥ n
- ❍ C. p ∥ q
- ❍ D. There are no parallel lines.

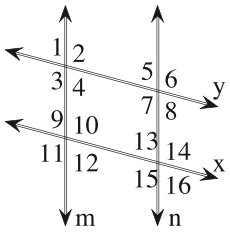
7. a) Writing If m∠3 = 127° and m∠6 = 127°, is line m parallel to line n?



- ❍ A. No, line m is not parallel to line n because ∠3 and ∠6 are congruent, but they are not alternate interior angles.
- ❍ B. Yes, line m is parallel to line n because ∠3 and ∠6 are supplementary alternate interior angles.
- ❍ C. Yes, line m is parallel to line n because ∠3 and ∠6 are congruent alternate interior angles.
- ❍ D. No, line m is not parallel to line n because ∠3 and ∠6 are alternate interior angles, but they are not congruent.

b) If line m is parallel to line n, what must be true about the relationship between ∠1 and ∠7? Explain.

8. a) Reasoning Which congruence statements justify m ∥ n? Check all that apply.

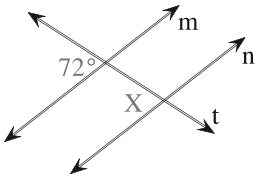


- ❑ A. If ∠9 ≅ ∠13, then m ∥ n because if corresponding angles are congruent, lines are parallel.
- ❑ B. If ∠4 ≅ ∠5, then m ∥ n because if alternate interior angles are congruent, lines are parallel.
- ❑ C. If ∠12 ≅ ∠13, then m ∥ n because if alternate interior angles are congruent, lines are parallel.
- ❑ D. If ∠5 ≅ ∠15, then m ∥ n because if corresponding angles are congruent, lines are parallel.
- ❑ E. If ∠10 ≅ ∠14, then m ∥ n because if alternate interior angles are congruent, lines are parallel.

b) Which congruence statements justify x || y? Check all that apply.

- ❑ A. If $\angle 9 \cong \angle 14$, then x || y because if corresponding angles are congruent, lines are parallel.
- ❑ B. If $\angle 1 \cong \angle 9$, then x || y because if corresponding angles are congruent, lines are parallel.
- ❑ C. If $\angle 3 \cong \angle 10$, then x || y because if alternate interior angles are congruent, lines are parallel.
- ❑ D. If $\angle 3 \cong \angle 11$, then x || y because if corresponding angles are congruent, lines are parallel.
- ❑ E. If $\angle 5 \cong \angle 13$, then x || y because if alternate interior angles are congruent, lines are parallel.

9. Error Analysis Your friend incorrectly says that line m is parallel to line n when the measure of angle X is 108°.



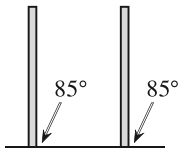
(The figure is not to scale.)

a) For which measure of angle X is line m parallel to line n?

b) What was your friend's likely mistake?

- ❍ A. Your friend used the same measure for angle X as that of its alternate interior angle.
- ❍ B. Your friend found the complement of 72°.
- ❍ C. Your friend used the same measure for angle X as that of its corresponding angle.
- ❍ D. Your friend found the supplement of 72°.

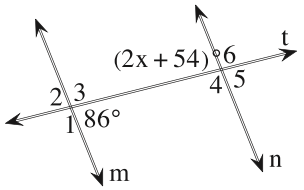
10. Architecture For safety reasons, a construction worker wants to make sure two studs for a wall are parallel. She measures the corresponding angles formed by the floor and the two studs. She finds that the measures of the angles are both 85°. If the studs are parallel, she can leave them as they are. Otherwise, they need to be fixed. Will the worker need to fix the studs?



(The figure is not to scale.)

- ❍ A. Yes, because the corresponding angles are not congruent.
- ❍ B. Yes, because the corresponding angles do not sum to 180°.
- ❍ C. No, because the corresponding angles sum to 180°.
- ❍ D. No, because the corresponding angles are congruent.

11. Multiple Representations Using alternate interior angles, write an equation in terms of x that will make line m parallel to line n.



(The figure is not to scale.)

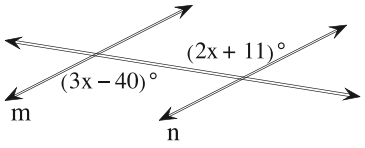
a) Which of the following equations will make line m parallel to line n?

- ❍ A. 2x + 54 = 86
- ❍ B. 2x – 126 = 94
- ❍ C. 2x + 54 = 94
- ❍ D. 2x – 126 = 86

b) Find the value of x that makes line m parallel to line n.

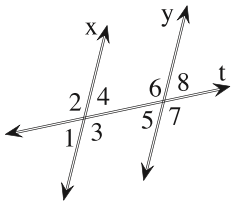
c) Find the measures of a different pair of angles that will make line m parallel to line n. Justify your reasoning.

12. Find the value of x for which m ∥ n.



(The figure is not to scale.)

13. Let m∠2 = 113° and m∠5 = 68°. Use alternate interior angles to decide if line x is parallel to line y.



(The figure is not to scale.)

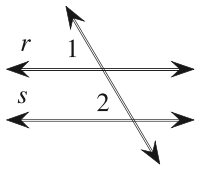
- ❍ A. Yes, because ∠4 and ∠6 are congruent.
- ❍ B. No, because ∠4 and ∠6 are not congruent.
- ❍ C. No, because ∠4 and ∠5 are not congruent.
- ❍ D. Yes, because ∠4 and ∠5 are congruent.

14. a) Challenge Find the value of x for which r ∥ s.

m∠1 = (63 – x)°

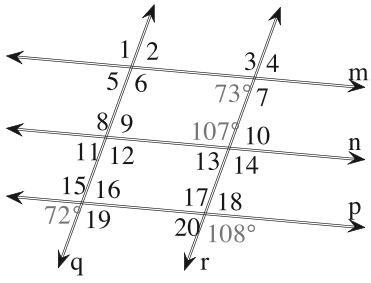
m∠2 = (72 – 2x)°

b) Find m∠1 and m∠2. Simplify your answer.



(The figure is not to scale.)

15. a) Challenge Determine which lines, if any, in the figure are parallel. Check all that apply.



- ❑ A. n ∥ p
- ❑ B. m ∥ n
- ❑ C. m ∥ p
- ❑ D. q ∥ r
- ❑ E. There are no parallel lines.

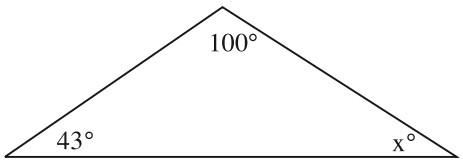
b) Explain your reasoning.

Name _____ Class _____ Date _____

Practice 25-3

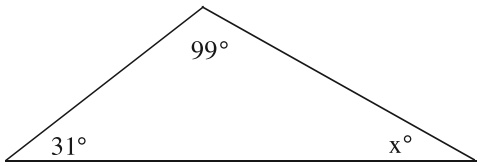
Interior Angles of Triangles

1. Find the number of degrees in the third angle of the triangle.



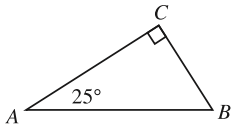
The figure is not drawn to scale.

2. An architect is designing a home. What is the measure of the missing angle of the roof?



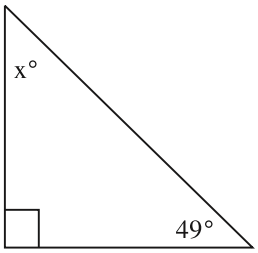
The figure is not drawn to scale.

3. Find the measure of angle B for the triangle shown.



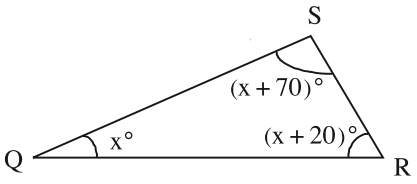
The figure is not drawn to scale.

4. There is a slide in the back of the school. The stairs for the slide go straight up. The angle made with the slide and the ground is 49°. What is the value of x?



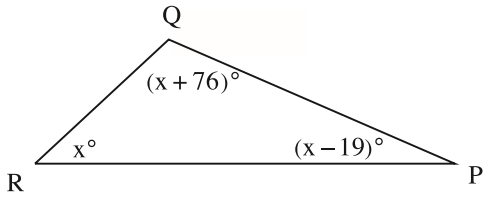
The figure is not drawn to scale.

5. In ΔQRS, $m\angle R$ is 20° more than $m\angle Q$ and $m\angle S$ is 70° more than $m\angle Q$. Find $m\angle R$.



The figure is not drawn to scale.

6. For the figure shown on the right, find the value of the variable, x, and the measures of angles ∠P, ∠Q, and ∠R.



The figure is not drawn to scale.

7. a) Writing If the measures of two angles of a triangle are 100° and 19°, what is the measure of the third angle?

b) Explain how a straight angle is related to the angles of a triangle.

8. Reasoning An art class is designing a sign to put by the entrance to the school. The sign is in the shape of a triangle and has one angle that is 87° and another which is 42°.

a) What is the measure of the third angle?

b) Explain how you could determine if the triangle is acute, right, or obtuse without finding the third angle.

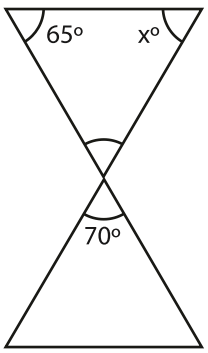
9. Error Analysis On a math test the students are given a right triangle. One of the acute angles has a measure of 55°. One student says that the measure of the other acute angle is 125°.

a) What is the measure of the other acute angle?

b) What error might the student have made?

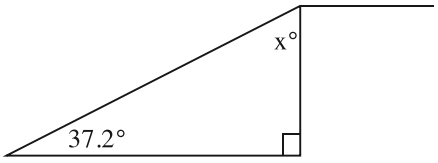
- ❍ A. The student only subtracted the right angle from 180°.
- ❍ B. The student subtracted the sum of the two given angles from 360°.
- ❍ C. The student added the right angle and the given acute angle, but did not subtract the sum from 180°.
- ❍ D. The student only subtracted the acute angle from 180°.

10. Statue A company is making different size statues that are in the shape of hour glasses. Use the figure to find the missing measure, x°.



The figure is not drawn to scale.

11. Estimation A ramp is being built to a building to help with deliveries. The angle that the bottom of the ramp makes with the ground is 37.2°.



The figure is not drawn to scale.

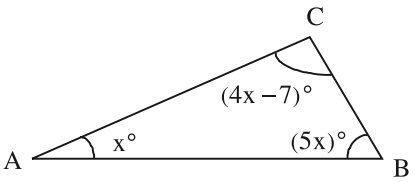
a) Estimate the measure of the other acute angle.

- ❍ A. 63°
- ❍ B. 48°
- ❍ C. 58°
- ❍ D. 53°

b) Find the exact measure of the other acute angle.

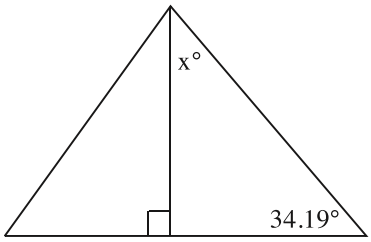
12. Mental Math If $m\angle B = 130°$ and $m\angle C = 10°$ for ΔABC, what is $m\angle A$?

13. In ΔABC, angle B is 5 times $m\angle A$ and angle C is 7° less than 4 times $m\angle A$. Find $m\angle B$. Simplify your answer.



The figure is not drawn to scale.

14. Challenge A pole in the middle of a tent is perpendicular to the ground. The measure of the angle made between one of the sides of the tent and the ground is 34.19°.



The figure is not drawn to scale.

a) What is the measure of the angle made between the side of the tent and the pole?

b) Explain how the measures of the acute angles will change if one of the angles is not a right angle.

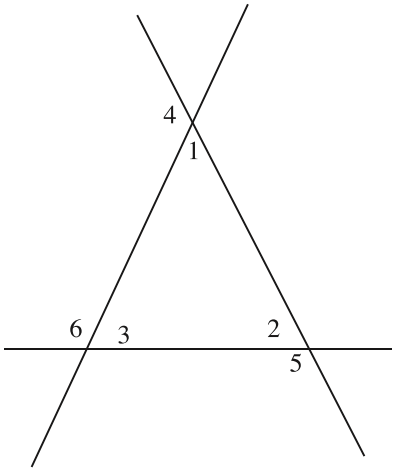
15. Challenge In ΔABC, $m\angle B$ is one-third the $m\angle A$ and $m\angle C$ is 37 less than the $m\angle A$. What are the measures of the angles of ΔABC?

Name _____ Class _____ Date _____

Practice 25-4

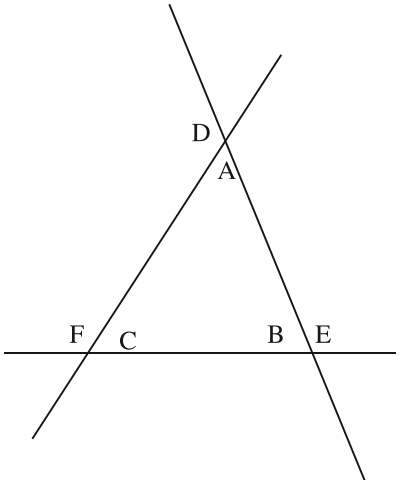
Exterior Angles of Triangles

1. Determine which of the labeled angles are exterior angles.



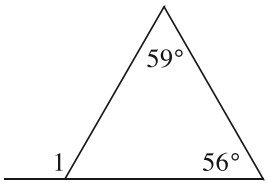
❍ A. ∠1, ∠5, and ∠6
❍ B. ∠4, ∠5, and ∠6
❍ C. ∠1, ∠2, and ∠3
❍ D. ∠3, ∠4, and ∠5

2. What are the two remote interior angles for ∠F?



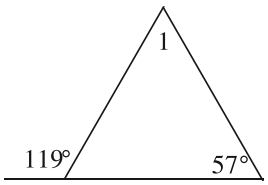
❍ A. ∠C and ∠B
❍ B. ∠A and ∠B
❍ C. ∠A and ∠C
❍ D. ∠E and ∠D

3. For the figure shown, find m∠1.



(The figure is not drawn to scale.)

4. For the figure shown, find m∠1.



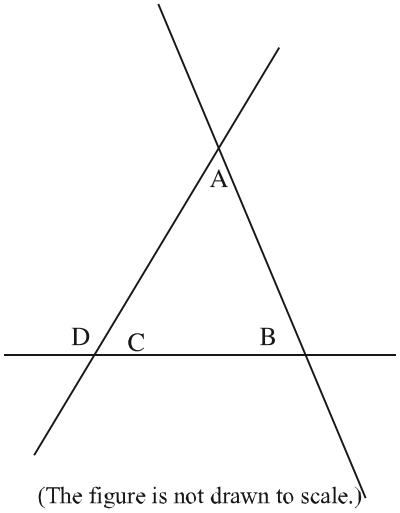
(The figure is not drawn to scale.)

5. Use the given information to find m∠A.

m∠D = 121°

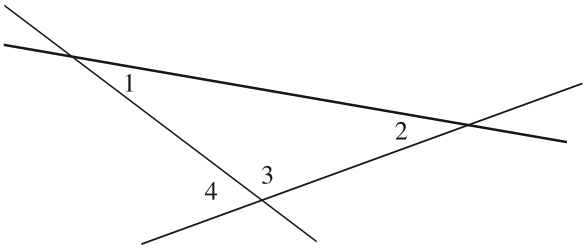
m∠A = (2x)°

m∠B = (x + 40)°



(The figure is not drawn to scale.)

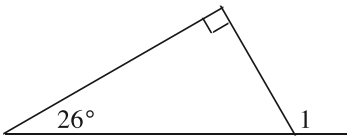
6. Given that m∠4 = 68°, m∠1 = (5x – 8)°, and m∠2 = (6x – 12)°, find m∠1 and m∠2.



(The figure is not drawn to scale.)

7. a) Writing For the figure shown, find m∠1.

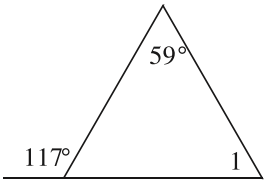
b) Explain two ways to find the missing angle measure of the triangle.



(The figure is not drawn to scale.)

8. a) Reasoning For the figure shown, find m∠1.

b) Can you find the measure of ∠1 without using an exterior angle and the other remote interior angle? Explain.

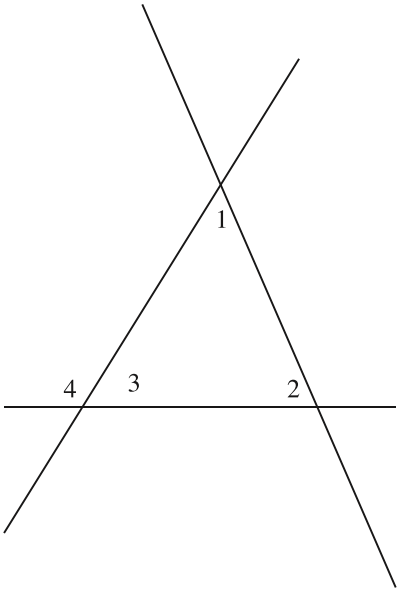


(The figure is not drawn to scale.)

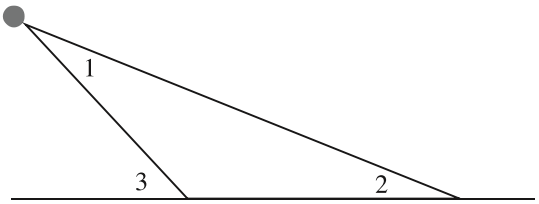
9. a) Error Analysis A student was asked to find $m\angle 1$ and $m\angle 2$ given that $m\angle 4 = 122°$, $m\angle 1 = (9x - 18)°$, and $m\angle 2 = (10x - 12)°$. He incorrectly said $m\angle 1 = 24°$ and $m\angle 2 = 34°$. Find $m\angle 1$ and $m\angle 2$.

b) What mistake might the student have made?

- ❍ A. He only solved for x.
- ❍ B. He thought the sum of $m\angle 1$, $m\angle 2$, and $m\angle 4$ was 180°.
- ❍ C. He thought the sum of $m\angle 1$ and $m\angle 2$ was 90°.

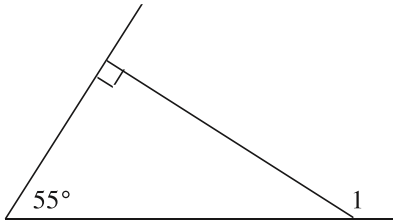


10. Airplane Spotting Two observers watch an airplane fly overhead. One observer looks up at $\angle 3$ to see the airplane, the other at $\angle 2$. If $m\angle 1 = (x + 28)°$, $m\angle 2 = (20x - 6)°$, and $m\angle 3 = 64°$, find $m\angle 1$.



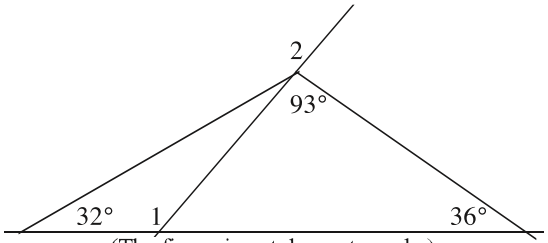
(The figure is not drawn to scale.)

11. Mental Math What is the measure of $\angle 1$?



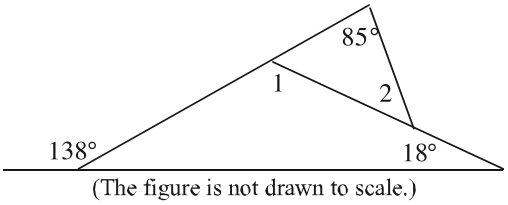
(The figure is not drawn to scale.)

12. For the figure shown, find $m\angle 1$ and $m\angle 2$.



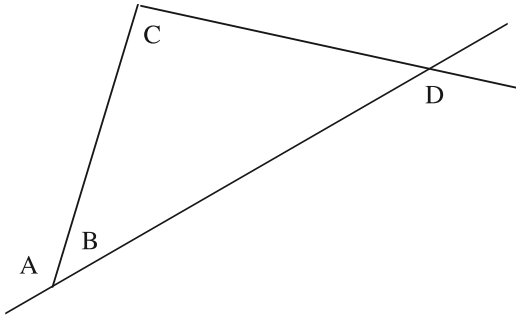
(The figure is not drawn to scale.)

13. For the figure shown, find $m\angle 1$ and $m\angle 2$.



(The figure is not drawn to scale.)

14. Challenge Given that $m\angle A = (16x)°$, $m\angle C = (8x + 21)°$, and $m\angle D = 129°$, what is $m\angle B$?



(The figure is not drawn to scale.)

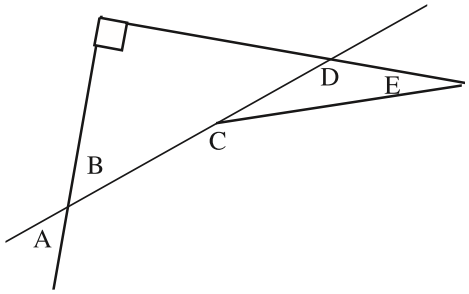
15. Challenge Use the information below to find $m\angle B$ and $m\angle E$.

$m\angle A = (9x - 71)°$

$m\angle C = (17x - 50)°$

$m\angle D = 127°$

$m\angle E = (y + 12)°$

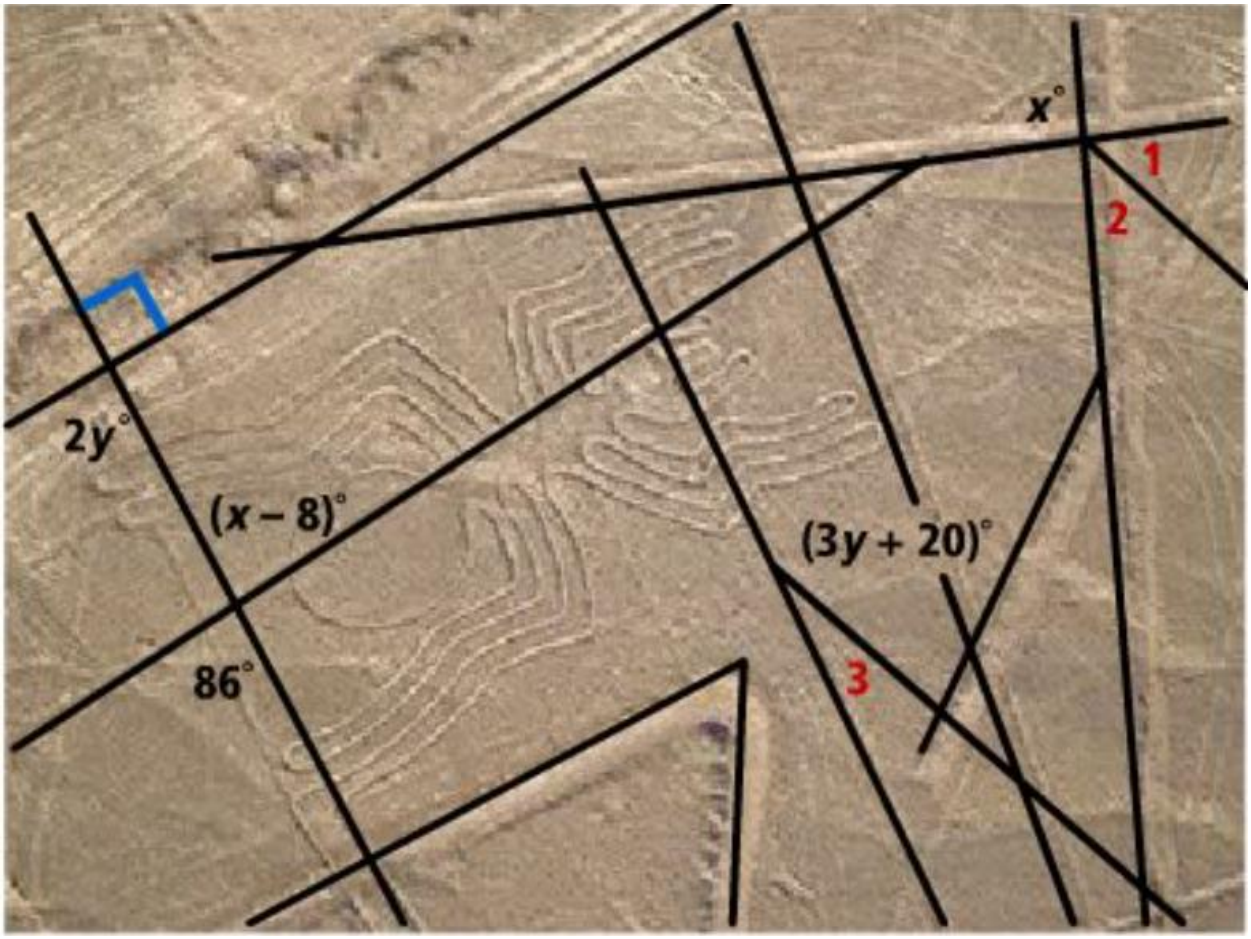


(The figure is not drawn to scale.)

Angles on the Plains of Nazca

Task

In the diagram below, assume that ∠1 and ∠3 complementary. What are the measures of ∠1, ∠2, and ∠3? Explain how you determined each angle.



How many squares are there on a standard checkerboard? The answer is not one, nor is it sixty-four. There are squares of many different sizes. Can you find the number of all of the different squares? Does a pattern exist that might help you solve this problem?