

digits 18-1 Compound Events

Part 1

Intro

In probability situations, an **action** is a process with an uncertain result. In the previous topic, you worked with actions that involved only one step or choice. Multi-step actions involve more than one step or choice.

Consider an example.

Action: Spin the spinner once.

A one-step action



Action: Spin the spinner twice.

A two-step action



Consider a different type of action.

Action: Choose one person from the group.

A one-step action



Action: Choose four people from the group.

A four-step action



continued on next page >

Compound Events - 1

Part 1

Intro continued

Here is another example.

Action: Roll one number cube.

A one-step action



Action: Roll three number cubes.

A three-step action



Finally, consider an action involving choosing clothes.

Action: Choose one shirt.

A one-step action



Action: Choose one shirt and one pair of pants.

A two-step action






Compound Events - 2

Part 1

Example Counting Choices for Actions

How many steps or choices does each action involve? Choose from the numerals below.

3 8 5 1 6 9 4 7 2

- Choose one fruit and one drink.

- Assign first, second, and third place ribbons.

- Choose a six-character password.


Solution

- a. Choose one fruit and one drink.

There are five items to pick from, but only two choices to make. The action has 2 steps.

- b. Assign first, second, and third place ribbons.

This action has 3 steps

- c. Choose a six-character password.

Each character requires a choice. This action has 6 steps.

Part 2

Intro

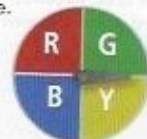
A **compound event** is an event associated with a multi-step action. A compound event is composed of events that are the outcomes of the steps of the action.

A compound event for a two-step action has two parts.

Action: Spin the spinner twice.

Compound event
Spin red. Then spin green.

(R, G)



Use parentheses to show the two outcomes are one compound event.

A compound event for a four-step action has four parts.

Action: Choose four people from a group.

Compound event
boy, girl, girl, boy

(B, G, G, B)



Parentheses show that the four outcomes are one compound event.

A compound event for a three-step action has three parts.

Action: Roll three number cubes.

Compound event
blue cube: 2, red cube: 6, green cube: 2

(2, 6, 2)



Parentheses show that the three outcomes are one compound event.

Part 2

Intro continued

A compound event for a two-step action has two parts.

Action: Choose a shirt and a pair of pants.

Compound event

yellow shirt and light blue jeans

(yellow, light blue)



Parentheses show that the two outcomes are one compound event.

Example Identifying Compound Events

Choose each group of jewelry on the right that shows a possible compound event for this action.

Action: Choose a necklace, a bracelet, and a ring.



Solution

These two groups show a possible outcome for each of the three steps of the action.



These two groups of jewelry are possible outcomes for the action of choosing a necklace, a bracelet, and a ring.

Compound Events - 5

Part 3

Intro

Two events are **independent events** if the occurrence of one event does not affect the probability of the other event.

Action: Choose one tile at random and replace it. Then choose a second tile at random.

Compound Event: Choose A. Then choose a B.



before choosing first tile

$$P(B) = \frac{2}{11} \quad (A, B)$$

after replacing first tile

$$P(B) = \frac{2}{11}$$

probability of 2nd choice not affected by 1st choice

The events *choose A* and *choose a B* are independent events for this action.

Two events are **dependent events** if the occurrence of the first event affects the probability of the second event.

Action: Choose one tile at random and set it aside. Then choose a second tile at random.

Compound Event: Choose A. Then choose a B.

before choosing first tile

$$P(B) = \frac{2}{11} \quad (A, B)$$

after setting aside the first tile

$$P(B) = \frac{2}{10} \text{ or } \frac{1}{5}$$

probability of 2nd choice affected by 1st choice

The events *choose A* and *choose a B* are dependent events for this action.

Compound Events - 6

Part 3

Example Classifying Events as Dependent or Independent



Decide if the events that make up each compound event are independent or dependent.

Action

Player 1 chooses a game piece.
Player 2 chooses a game piece.

Compound Event

Player 1 chooses red.
Player 2 chooses green.

Action

Spin the spinner twice.

Compound Event

The spinner stops on green twice.

Solution

Compound Event

Player 1 chooses red.
Player 2 chooses green.

Before Player 1 chooses

$$P(\text{choose green}) = \frac{1}{4}$$

After Player 1 chooses

$$P(\text{choose green}) = \frac{1}{3}$$

The probability changes.

The outcome of Player 1's choice affects the probability that Player 2 chooses green. The events are dependent.

Compound Event

The spinner stops on green twice.

Before first spin

$$P(\text{spin green}) = \frac{1}{4}$$

After first spin

$$P(\text{spin green}) = \frac{1}{4}$$

The probability does not change.

The outcome of the first spin does not affect the probability of spinning green on the second spin. The events are independent.

Key Concept

A compound event is an event associated with a multi-step action.

Events are independent if the occurrence of one event does not affect the probability of the other event.

Action: Toss a coin. Roll a number cube.

Step 1 Outcome

Step 2 Outcome



compound event tails, 3
(T, 3)

The outcomes are independent events because tossing tails does not affect the probability of spinning 3.

Events are dependent if the occurrence of the first event affects the probability of the second event.

Action: Choose two people for your group.

Step 1 Outcome

Step 2 Outcome



compound event boy, girl
(B, G)

The outcomes are dependent events because choosing a boy affects the probability of choosing a girl.

digits 18-2 Sample Spaces

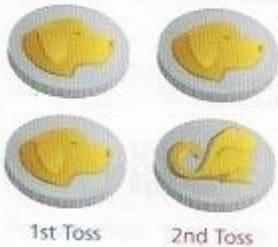
Part 1

Intro

To write the sample space of a multi-step action, use an organized approach to list all possible outcomes of the action.

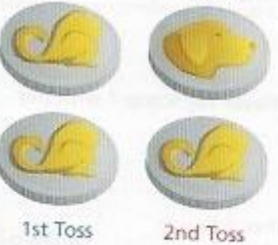
Action: Toss a coin twice.

Start with one outcome of the first toss, heads, and pair it with each possible outcome of the second toss, heads and tails.



Sample space: Possible outcomes
(Heads, Heads) (H, H)
(Heads, Tails) (H, T)

Then, use the other outcome of the first toss, tails, and pair it with each possible outcome of the second toss, heads and tails.



Sample space: Possible outcomes
(Tails, Heads) (T, H)
(Tails, Tails) (T, T)

Example Using Organized Lists for Sample Spaces

For each condition, use an organized list to show all possible flavor combinations for Scoop 1 and Scoop 2.

- Scoop 1 and Scoop 2 can be either the same flavor or different flavors.
- Scoop 1 and Scoop 2 must be different flavors.



Sample Spaces - 1

Part 1

Example continued

Solution

- The scoops can be the same or different flavors, so pair each flavor for Scoop 1 with each flavor to Scoop 2.

| | Scoop 1 | Scoop 2 |
|------------|------------|------------|
| 1st flavor | peach | peach |
| | peach | strawberry |
| | peach | banana |
| 2nd flavor | strawberry | peach |
| | strawberry | strawberry |
| | strawberry | banana |
| 3rd flavor | banana | peach |
| | banana | strawberry |
| | banana | banana |

You can also write the organized list of the sample space using a capital letter for each flavor.

P = peach, S = strawberry, B = banana

Sample Space (Scoop 1, Scoop 2)

(P, P) (S, P) (B, P)
(P, S) (S, S) (B, S)
(P, B) (S, B) (B, B)

- The scoops must be different flavors, so pair each flavor for Scoop 1 with the other two flavors for Scoop 2.

| | Scoop 1 | Scoop 2 |
|------------|------------|------------|
| 1st flavor | peach | strawberry |
| | peach | banana |
| 2nd flavor | strawberry | peach |
| | strawberry | banana |
| 3rd flavor | banana | peach |
| | banana | strawberry |

You can also write the organized list of the sample space using a capital letter for each flavor.

P = peach, S = strawberry, B = banana

Sample Space (Scoop 1, Scoop 2)

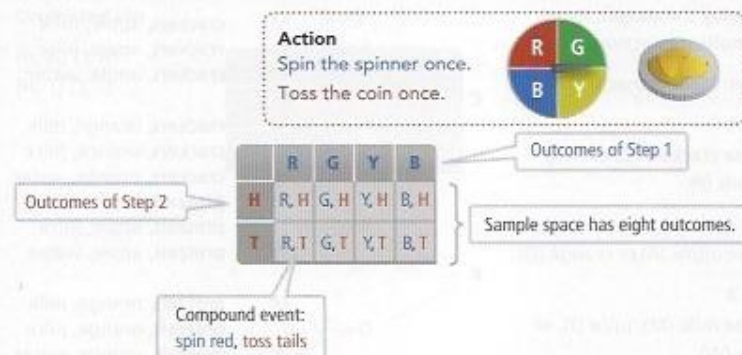
(P, S) (S, P) (B, P)
(P, B) (S, B) (B, S)

Sample Spaces - 2

Part 2

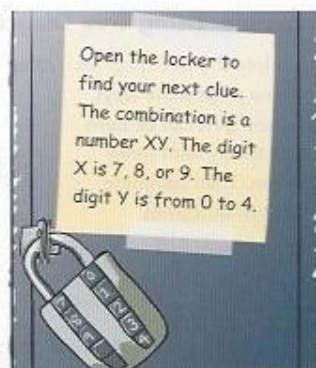
Intro

You can use a table to display the sample space of a two-step action.



Example Using Tables for Sample Spaces

On a scavenger hunt, a clue leads you to a locker with a note on it. Use a table to show all possible combinations of the lock. How many combinations are possible?



Solution

There are 15 possible lock combinations.

Possible Lock Combinations

| | 7 | 8 | 9 |
|---|----|----|----|
| 0 | 70 | 80 | 90 |
| 1 | 71 | 81 | 91 |
| 2 | 72 | 82 | 92 |
| 3 | 73 | 83 | 93 |
| 4 | 74 | 84 | 94 |

The first digit is 7, 8, or 9.

The second digit is from 0 to 4.

Sample Spaces - 3

Part 3

Intro

You can use a tree diagram to display the sample space of a multi-step action.

Action: Make a snack.

Step 1

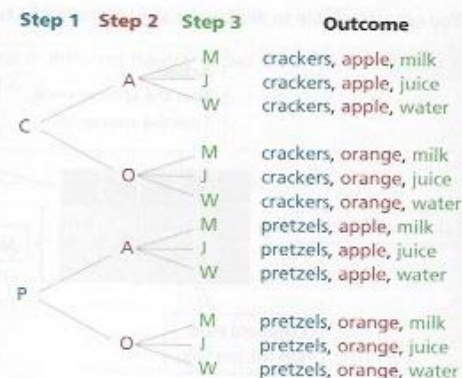
Choose crackers (C) or pretzels (P).

Step 2

Choose apple (A) or orange (O).

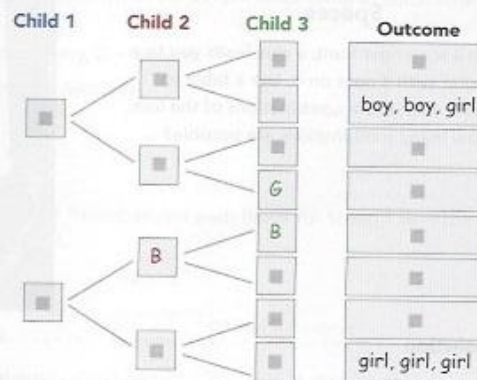
Step 3

Choose milk (M), juice (J), or water (W).

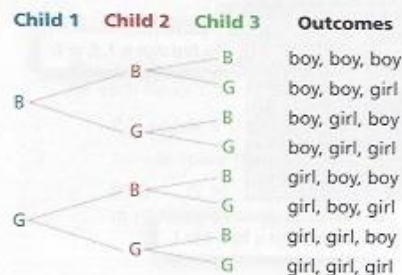


Example Using Tree Diagrams for Sample Spaces

Complete the tree diagram to show all possible ways boys and girls can be born into a family with three children. Then complete the list of outcomes.



Solution



Sample Spaces - 4

Key Concept

These displays show three ways to represent the sample space for tossing two coins.

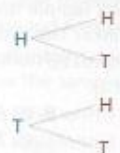
Organized List

(H, H) (T, H)
(H, T) (T, T)

Table

| | Heads | Tails |
|-------|-------|-------|
| Heads | H, H | T, H |
| Tails | H, T | T, T |

Tree Diagram



Sample Spaces - 5

digits 18-3 Counting Outcomes

Key Concept

The tree diagram of the sample space for this action illustrates the Counting Principle.

Action

Choose one shape:



Choose one color: orange, purple, green

Sample Space

2 outcomes Shape



3 outcomes Color

6 outcomes
 $2 \times 3 = 6$

The Counting Principle

If there are m possible outcomes of one action and n possible outcomes of a second action, then there are $m \times n$ outcomes of the first action followed by the second action.

Part 1

Example Using the Counting Principle

Use the Counting Principle to find the number of possible outcomes of the class election. Then check your result by writing out the sample space for the election.

Ballot for Class Election

President Marta ☐

Ethan ☐

Secretary Ben ☐

Rosa ☐

Zack ☐

Treasurer Andi ☐

Theo ☐

Counting Outcomes - 1

on next page >

Part 1

Example continued

Solution

You can extend the Counting Principle to three or more actions.

$$\begin{array}{c} \text{President} \\ 2 \text{ choices} \end{array} \times \begin{array}{c} \text{Secretary} \\ 3 \text{ choices} \end{array} \times \begin{array}{c} \text{Treasurer} \\ 2 \text{ choices} \end{array} = 12$$

There are 12 possible outcomes for the election.

Check

Use an organized list to show the 12 possible outcomes in the sample space.

| | President | Secretary | Treasurer |
|----|-----------|-----------|-----------|
| 1 | Marta | Ben | Andi |
| 2 | Marta | Ben | Theo |
| 3 | Marta | Rosa | Andi |
| 4 | Marta | Rosa | Theo |
| 5 | Marta | Zack | Andi |
| 6 | Marta | Zack | Theo |
| 7 | Ethan | Ben | Andi |
| 8 | Ethan | Ben | Theo |
| 9 | Ethan | Rosa | Andi |
| 10 | Ethan | Rosa | Theo |
| 11 | Ethan | Zack | Andi |
| 12 | Ethan | Zack | Theo |

Part 2

Intro

Given the sample space of an action, you can identify outcomes that have particular characteristics. These outcomes compose an event in the sample space.

Action

Toss a coin and roll a standard number cube.

Sample space

(H, 1) (H, 2) (H, 3)
(H, 4) (H, 5) (H, 6)
(T, 1) (T, 2) (T, 3)
(T, 4) (T, 5) (T, 6)

Event

Toss tails or roll an odd number.

(T, 1) (T, 2) (T, 3)
(T, 4) (T, 5) (T, 6)
(H, 1) (H, 3) (H, 5)

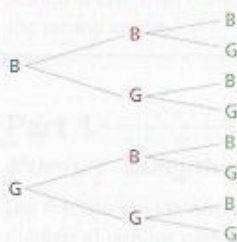
"Or" means outcomes can include a T, an odd number, or both.

9 outcomes in the event

Example Counting Outcomes with Tree Diagrams

Use the tree diagram that shows all possible ways boys (B) and girls (G) can be born into a family with three children. Complete the table.

Child 1 Child 2 Child 3



| Event | Outcomes in Event | Number of Outcomes |
|----------------------|-------------------|--------------------|
| Get exactly 2 boys | | |
| Get at least 2 girls | | |

Solution

Event: Get exactly two boys.

Child 1 Child 2 Child 3



Outcomes in Event: 3

(boy, boy, girl)

(boy, girl, boy)

(girl, boy, boy)

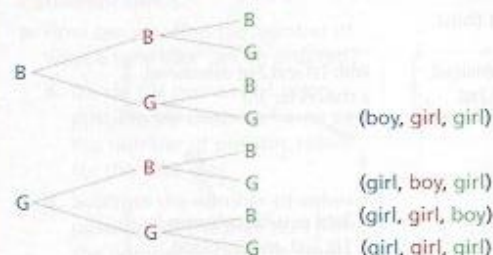
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Part 2

Solution continued

Event: Get at least two girls.

Child 1 Child 2 Child 3 Outcomes in Event: 4



| Event | Outcomes in Events | Number of Outcomes |
|----------------------|---|--------------------|
| Get exactly 2 boys | (boy, boy, girl), (boy, girl, boy), (girl, boy, boy) | 3 |
| Get at least 2 girls | (boy, girl, girl), (girl, boy, girl), (girl, girl, boy), (girl, girl, girl) | 4 |

Part 3

Example Counting Outcomes in Events

In how many different ways can Alice, Becky, Carl, and David place first, second, and third in a race? In how many different ways can Alice finish ahead of David?

Counting Outcomes - 4

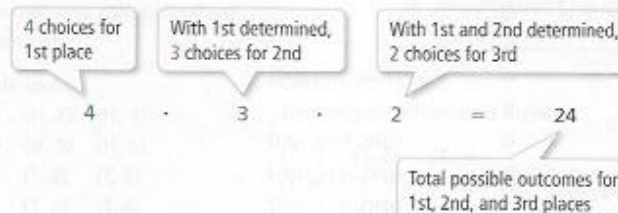
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Part 3

Example continued

Solution

You can use the Counting Principle to find the number of ways the four runners can place first, second, and third.



You can make an organized list of the possible outcomes for the first three places. Use the list to find and count all possible outcomes in which A (Alice) comes before D (David), or A appears and D does not.

| | | | |
|-----|-----|-----|-----|
| ABC | BAC | CAB | DAB |
| ABD | BAD | CAD | DAC |
| ACB | BCA | CBA | DBA |
| ACD | BCD | CBD | DBC |
| ADB | BDA | CDA | DCA |
| ADC | BDC | CDB | DCB |

There are 24 different ways the runners can place first, second, and third in the race. There are 12 different ways Alice can finish ahead of David.

Counting Outcomes - 5

Unit 4

4/20/20

Compound Events

- Begin on a new page
- Write the date and unit in the top corners of the page
- Write the title across the top line

Independent Events

Two events are independent events if the occurrence of one event does not affect the probability of the other event.

Example

You flip a coin twice. Getting heads on the first flip does not affect the probability of heads on the second flip.



Compound Event

An event composed of more than one action.

Example

You roll a number cube. Then, you roll it again.
Or, you roll 2 number cubes.



Dependent Events

Two events are dependent events if the occurrence of the first event affects the probability of the second event.

Example

Two students are chosen at random to win a prize. After choosing the first one, there are fewer students to pick from, so your chance of getting chosen increases.



$$P(\text{prize}) = \frac{1}{5}$$



$$P(\text{prize}) = \frac{1}{4}$$

Unit 4

4/20/20

Counting Outcomes

- Begin on a new page
- Write the date and unit in the top corners of the page
- Write the title across the top line

Example 1

You must choose a four digit password.

Number of possible passwords: $10 \times 10 \times 10 \times 10 = 10,000$

Note that each choice was Independent of the previous choice.

Counting Principle

If there are m possible outcomes for one action and n possible outcomes for a second action, then there are $m \times n$ possible outcomes for the first action followed by the second action.

Example 2

You must choose a four digit password, but digits may not repeat.

Number of possible passwords: $10 \times 9 \times 8 \times 7 = 5,040$

Nine choices,
because you may not
repeat the first digit.

Eight choices,
because you may not
repeat either of the
first two digits.

Seven choices,
because you may not
repeat any of the first
three digits.

Note that each choice was Dependent on the previous choice.

**Practice
17-1****Compound Events**

1. Roll three number cubes. How many steps or choices does the action involve?
2. Which action involves 2 steps?
 - ☐ A. Choose four people from a group.
 - ☐ B. Choose a book from the library.
 - ☐ C. Choose two books from the library.
 - ☐ D. Roll three number cubes.
3. Roll a number cube two times. Find a compound event for the action.
 - ☐ A. (1, 5, 3, 5)
 - ☐ B. (1, 1, 1)
 - ☐ C. (1, 5)
 - ☐ D. 1
4. Roll a number cube four times. Find a compound event for the action.
 - ☐ A. (4, 4, 4, 4)
 - ☐ B. (2, 2, 2)
 - ☐ C. (3, 5, 4, 2, 1)
 - ☐ D. (3, 2)
5. Which compound event is composed of independent events?

| Action | Compound Event |
|--|----------------|
| Choosing three numbers from 1 to 10 | (8, 5, 4) |
| Choosing three people from a group of two girls and two boys | (B, B, G) |

- ☐ A. (8, 5, 4) and (B, B, G)
 - ☐ B. (8, 5, 4) only
 - ☐ C. (B, B, G) only
 - ☐ D. None of the above
6. Which compound event is composed of dependent events?

| Action | Compound Event |
|---|----------------|
| Choosing any two colored cards from a stack, replacing the card each time you choose. | (Red, Red) |
| Choosing any two colored cards from a stack, without replacing the card each time you choose. | (Red, Green) |

- ☐ A. (Red, Red) and (Red, Green)
- ☐ B. (Red, Red) only
- ☐ C. (Red, Green) only
- ☐ D. None of the above

7. **Writing** The number on your soccer jersey is 23. You decide to choose a three-character password by selecting at random two different letters, followed by one number, from the phrase SOCCER 23.
- a) Which choice shows a compound event for this action?
- ☐ A. (O, R, 2, 3)
 - ☐ B. (2, O, R)
 - ☐ C. (O, R, 3)
 - ☐ D. (O, 2, R)
- b) Write your own multi-step action. Then write a possible compound event for that action.
8. a) **Reasoning** Are the events that make up the following compound event independent or dependent?
- Action:**
Spin the spinner. Then spin again.
- Compound Event:**
The spinner lands on green. The spinner then lands on yellow.
- b) Is it possible to have the same outcome for both parts of a two-step dependent compound event? Explain.
9. **Error Analysis** On a recent math test students were asked to find the number of steps in rolling three number cubes three times. Fausto gave an incorrect answer, 3.
- a) Find the number of steps in the action.
- b) Which error might Fausto have made?
- ☐ A. Fausto should have added the number of rolls to the number of number cubes to get the correct answer. He instead used the number of rolls as the number of steps.
 - ☐ B. Fausto should have subtracted the number of rolls from the number of number cubes to get the correct answer. He instead used the number of rolls as the number of steps.
 - ☐ C. Fausto should have subtracted the number of rolls from the number of number cubes to get the correct answer. He instead used the number of number of cubes as the number of steps.
 - ☐ D. Fausto should have multiplied the number of rolls by the number of number cubes to get the correct answer. He instead used the number of number cubes as the number of steps.

- 10. Ice Cream** You and a friend went to get ice cream. You both want three scoops but you cannot decide on which flavors. You tell the salesperson to surprise you with any combination of the flavors shown. Your friend also tells the salesperson to make it a surprise but also says not to use the same flavor twice. Whose action produces a compound event which is composed of independent events?

| Flavors | Action | Compound Event |
|---|---|----------------|
| Chocolate (C) Strawberry (S) Vanilla (V) | You order any 3 flavors. | (P, S, P) |
| Mocha (M) Pistachio (P) | Your friend orders any 3 different flavors. | (S, P, C) |

- ☐ A. both of yours ☐ C. only yours
☐ B. only your friend's ☐ D. neither of yours
- 11. Multiple Representations** Your teacher asks you to make 4 three-letter words using the letters from the word MATHEMATICS.
- a) Which choice shows a compound event for this action?
- ☐ A. (MAT, MEAT, EAT, SET) ☐ C. (MAT, MATH, MEAT)
☐ B. (MAT, MATH, HAT) ☐ D. (MAT, HAT, EAT, SET)
- b) Write another compound event for this action.
- 12.** A person can order a new car with a choice of 6 possible colors, with or without air conditioning, with or without automatic transmission, with or without power windows, and with or without a CD player. How many steps are needed to purchase a car if you select one option from each category?
- 13.** Your options for purchasing a cell phone are shown in the table. Which of the events shown is a compound event composed of independent events?

| Action | Options | Event |
|---------------------------------------|---|--------|
| Choose one color. | Red(1), Blue (2), Black (3), White (4), Silver (5) | 1 |
| Choose two different ring tones. | 1 Beep (1), 2 Beeps (2), 3 Beeps (3), 4 Beeps (4), 5 Beeps (5) | (5, 4) |
| Select the number of monthly minutes. | 500, 600, 700, 750, 800, 900 | 900 |

- ☐ A. (5, 4) ☐ C. 900
☐ B. 1 ☐ D. None of the above.

- 14. Challenge** You are asked to make three three-letter words using the letters from the word PROBABILITY. (BAR, BOY, LIT) represents a possible outcome.

| Action | Compound Event |
|---|-----------------|
| Make three words using each letter only once. | (BAR, BOY, LIT) |
| Make three words replacing the letter each time you choose. | (BAR, BOY, LIT) |
| Make one word using each letter only once. Do this three times. | (BAR, BOY, LIT) |
| Make one word replacing the letter each time you choose. Do this three times. | (BAR, BOY, LIT) |

- a) For which of the actions shown is this compound event composed of independent events? Check all that apply.
- ☐ A. Make one word replacing the letter each time you choose. Do this three times.
 - ☐ B. Make three words replacing the letter each time you choose.
 - ☐ C. Make one word using each letter only once. Do this three times.
 - ☐ D. Make three words using each letter only once.
- b) Describe in words what you think the probability is of producing this compound event for each action.
- 15. Challenge** Your teacher asks you to make three three-letter words using the letters from the word MATHEMATICS. To have a compound event composed of dependent events, you can only use each letter once to make all three words.
- a) Which compound event is composed of dependent events?
- ☐ A. (MAT, EAT, HAT)
 - ☐ B. (ATE, HAT, SIT)
 - ☐ C. (MAT, HEM, SAT)
 - ☐ D. (MAT, HAM, CAT)
- b) Write another compound event composed of dependent events for this action.

**Practice
17-2****Sample Spaces**

1. Two friends at a restaurant each order a fruit drink. The available flavors are kiwi (K), orange (O), or watermelon (W). Which list represents the sample space of the friends' fruit drinks? The lists are written in the format (Friend 1, Friend 2).
- ☐ A. (K, O)
 (K, W)
 (O, K)
 (O, W)
 (W, K)
 (W, O)
- ☐ B. (K, K)
 (K, O)
 (K, W)
 (O, K)
 (O, O)
 (O, W)
 (W, K)
 (W, O)
 (W, W)
- ☐ C. (K, O, W)
 (K, W, O)
 (K, K, K)
 (O, K, W)
 (O, W, K)
 (O, O, O)
 (W, K, O)
 (W, O, K)
 (W, W, W)
- ☐ D. (K, O, W)
 (K, W, O)
 (O, K, W)
 (O, W, K)
 (W, K, O)
 (W, O, K)
2. Three friends at a restaurant each order a different flavored fruit drink. The available flavors are strawberry (S), peach (P), and orange (O). Which list represents the sample space of the friends' fruit drinks? The lists are written in the format (Friend 1, Friend 2, Friend 3).
- ☐ A. (S, P, O)
 (S, O, P)
 (P, S, O)
 (P, O, S)
 (O, S, P)
 (O, P, S)
- ☐ B. (S, S)
 (S, P)
 (S, O)
 (P, S)
 (P, P)
 (P, O)
 (O, S)
 (O, P)
 (O, O)
- ☐ C. (S, P, O)
 (S, O, P)
 (S, S, S)
 (P, S, O)
 (P, O, S)
 (P, P, P)
 (O, S, P)
 (O, P, S)
 (O, O, O)
- ☐ D. (S, P, O)
 (P, O, S)
 (O, S, P)
3. Use a table to show the sample space of two-digit numbers using the digits 8, 3, 2, 4. Use the column label as the tens digit and the row label as the ones digit to complete the table.

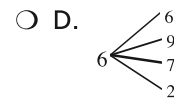
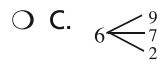
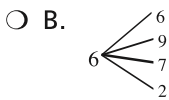
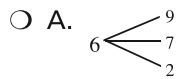
| Sample Space | | | | |
|--------------|-------|-------|-------|-------|
| | 8 | 3 | 2 | 4 |
| 8 | _____ | _____ | _____ | _____ |
| 3 | _____ | _____ | _____ | _____ |
| 2 | _____ | _____ | _____ | _____ |
| 4 | _____ | _____ | _____ | _____ |

4. a) Complete the table to show the sample space of number-letter combinations using the digits in the number 9,825 and the letters in the word GAME.

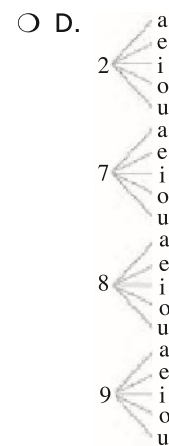
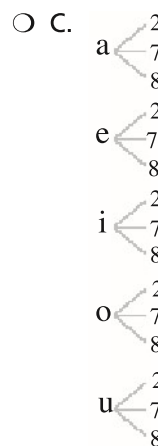
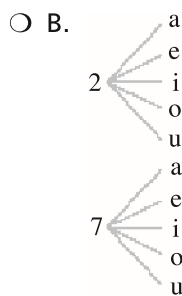
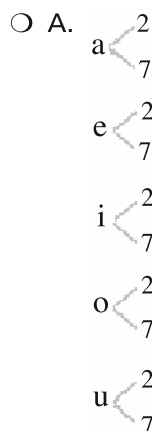
| Sample Space | | | | |
|--------------|-------|-------|-------|-------|
| | 9 | 8 | 2 | 5 |
| G | _____ | _____ | _____ | _____ |
| A | _____ | _____ | _____ | _____ |
| M | _____ | _____ | _____ | _____ |
| E | _____ | _____ | _____ | _____ |

- b) Find the number of possible outcomes.

5. Which tree diagram displays the sample space for choosing two different digits from the number 6,972? The order in which the digits are selected is important. For example, choosing 6 and then 9 is not the same selection as choosing 9 and then 6.



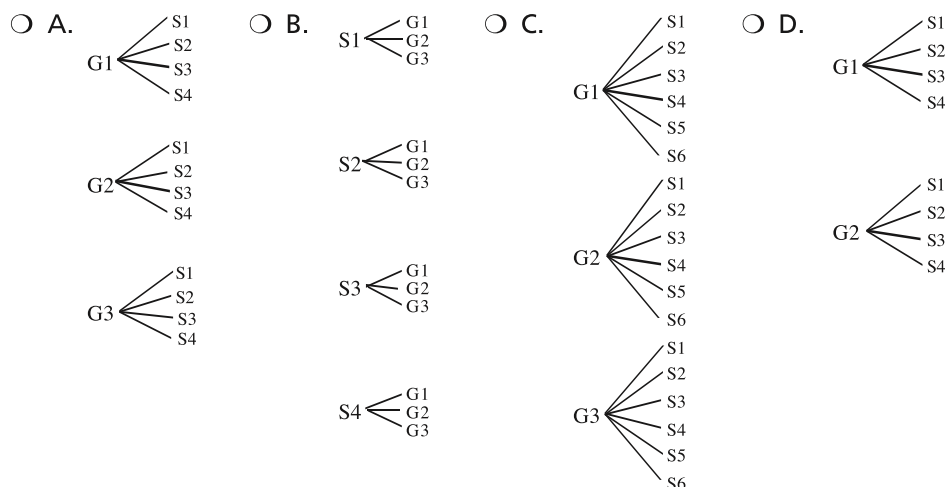
6. a) Which tree diagram displays the sample space for choosing a vowel (a, e, i, o, u) and then a number (2 or 7)?



- b) How many possible outcomes are there?

7. Writing There are four stores that sell school supplies (S1, S2, S3, and S4) and three stores that sell sporting goods (G1, G2, G3).

a) Which tree diagram displays the sample space of stores you could visit to buy a tennis racquet and then a backpack?



b) How many different outcomes are there?

c) Do you see a relationship between the number of possible outcomes of each step and the number of outcomes in the sample space? Explain.

8. Reasoning A soccer tournament assigns a unique two-color uniform to each team using the colors yellow (Y), green (G), orange (O), and purple (P). Each uniform is mostly one color with a different colored stripe.

a) Which list represents the sample space? The lists are written in the format (main color, stripe color).

- | | | | |
|-------------------|-------------|-------------|-------------|
| ○ A. (Y, G, O, P) | ○ B. (Y, G) | ○ C. (Y, G) | ○ D. (Y, G) |
| (G, O, P, Y) | (Y, O) | (Y, O) | (Y, O) |
| (O, P, Y, G) | (Y, P) | (Y, P) | (Y, P) |
| (P, Y, G, O) | (G, O) | (G, Y) | (G, Y) |
| | (G, P) | (G, O) | (G, G) |
| | (O, P) | (G, P) | (G, O) |
| | | (O, Y) | (G, P) |
| | | (O, G) | (O, Y) |
| | | (O, P) | (O, G) |
| | | (P, Y) | (O, O) |
| | | (P, G) | (O, P) |
| | | (P, O) | (P, Y) |
| | | | (P, G) |
| | | | (P, O) |

b) Does it matter what order you list the different ways to assign the uniform colors? Explain your reasoning.

9. Error Analysis A clothing store sells shirts with long, short, or no sleeves. Each style is available in gray, blue, or pink. A clerk incorrectly states the sample space for the possible color and sleeve styles as (gray, long), (gray, short), (blue, long), (blue, short), (pink, long), and (pink, short).

- a) Complete the table that represents the sample space of color and sleeve styles combinations.

| Sample Space | | | |
|--------------|--------------|--------------|--------------|
| | gray | blue | pink |
| long | _____, _____ | _____, _____ | _____, _____ |
| short | _____, _____ | _____, _____ | _____, _____ |
| no | _____, _____ | _____, _____ | _____, _____ |

- b) What error did the clerk make?
- ☐ A. The clerk left out the possible outcomes for shirts with no sleeves.
 - ☐ B. The clerk left out the possible outcomes for gray colored shirts.
 - ☐ C. The clerk left out the possible outcomes for shirts with short sleeves.
 - ☐ D. The clerk left out the possible outcomes for pink colored shirts.

10. **Baking** A bakery sells wheat, multi-grain, rye, and oat bread. Each type of bread is available as a round loaf or as dinner rolls. Use a table to show the sample space for the type and style of bread.

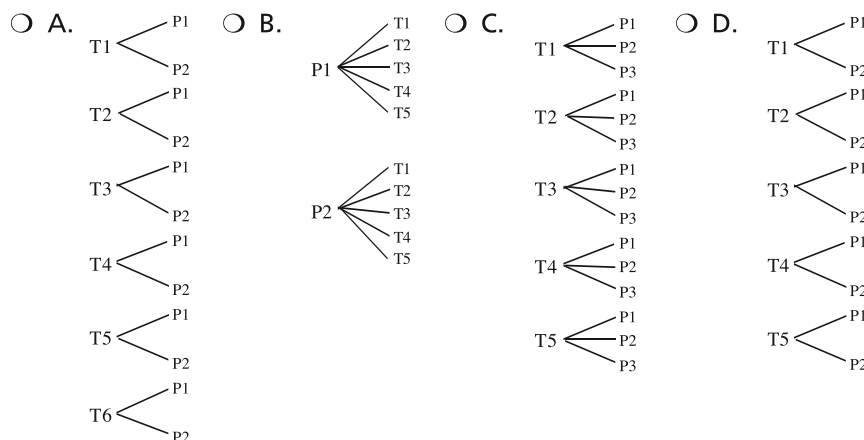
- a) Complete the table.

| Sample Space | | |
|--------------|--------------|--------------|
| | Loaf | Rolls |
| Wheat | _____, _____ | _____, _____ |
| Multi-grain | _____, _____ | _____, _____ |
| Rye | _____, _____ | _____, _____ |
| Oat | _____, _____ | _____, _____ |

- b) Find the number of possible outcomes.

11. **Multiple Representations** A designer has designed two pairs of pants (P1 and P2) and five tops (T1, T2, T3, T4, and T5) to create outfits.

- a) Which tree diagram displays the sample space of possible outfits if she chooses a top and then pants for each outfit?



- b) How many different outfits could she create?
- c) Use a table to display the same sample space and compare the table to your tree diagram.

12. Three friends choose something to drink. The available choices are iced tea (T), milk (M), fruit punch (P), and lemonade (L).

a) Which list represents the sample space if each friend chooses a different drink? The lists are written in the format (Drink 1, Drink 2, Drink 3).

☐ A. (T, M, P)

(T, M, L)

(T, P, L)

(M, T, P)

(M, T, L)

(M, P, L)

(P, T, M)

(P, T, L)

(P, M, L)

(L, T, M)

(L, T, P)

(L, M, P)

☐ B. (T, M, P)

(T, M, L)

(T, P, L)

(M, T, L)

(M, P, L)

☐ C. (T, M, P)

(T, M, L)

(T, P, M)

(T, P, L)

(T, L, M)

(T, L, P)

(M, T, P)

(M, T, L)

(M, P, T)

(M, P, L)

(M, L, T)

(M, L, P)

(P, T, M)

(P, T, L)

(P, M, T)

(P, M, L)

(P, L, T)

(P, L, M)

(L, T, M)

(L, T, P)

(L, M, T)

(L, M, P)

(L, P, T)

(L, P, M)

b) Which list below represents the sample space if the first two friends choose the same drink and the third friend chooses a different drink? The lists are written in the format (Drink 1, Drink 2, Drink 3).

☐ A. (T, T, M)

(T, T, P)

(T, T, L)

☐ B. (T, T, M)

(M, M, T)

(P, P, T)

(L, L, T)

☐ C. (T, T, M)

(T, T, P)

(T, T, L)

(M, M, T)

(M, M, P)

(M, M, L)

(P, P, T)

(P, P, M)

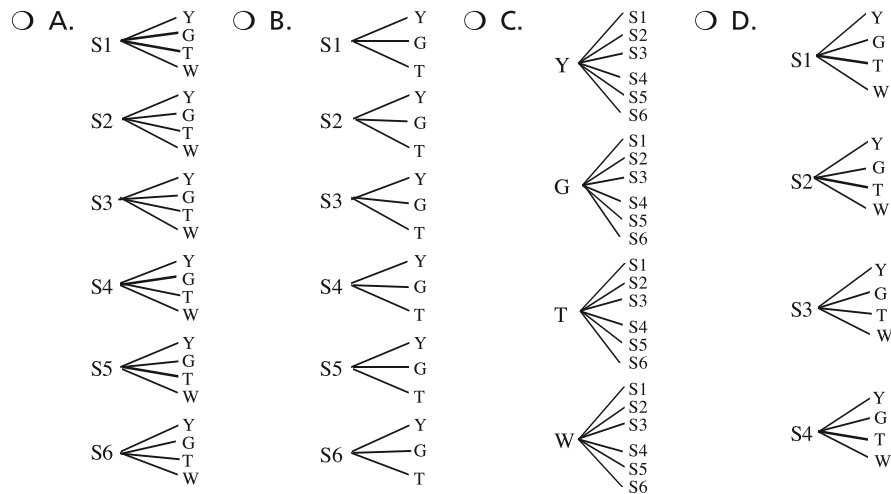
(P, P, L)

(L, L, T)

(L, L, M)

(L, L, P)

13. Cassandra needs to buy a house for her dog. She can choose from six different styles (S1, S2, S3, S4, S5, or S6). Each style comes in yellow (Y), green (G), tan (T), or white (W). Which tree diagram displays the sample space if she chooses a style and then a color?



14. **Challenge** Vincent forgot the last two digits for his bicycle lock. He remembers that each digit is 1 through 5.

a) Which table shows the sample space if each digit is 1 through 5?

○ A.

| | | | | | |
|---|----|----|----|----|----|
| | 1 | 2 | 3 | 4 | 5 |
| 1 | 11 | 21 | 31 | 41 | 51 |
| 2 | 12 | 22 | 32 | 42 | 52 |
| 3 | 13 | 23 | 33 | 43 | 53 |
| 4 | 14 | 24 | 34 | 44 | 54 |
| 5 | 15 | 25 | 35 | 45 | 55 |

○ B.

| | | | | | | |
|---|----|----|----|----|----|----|
| | 0 | 1 | 2 | 3 | 4 | 5 |
| 0 | 00 | 10 | 20 | 30 | 40 | 50 |
| 1 | 01 | 11 | 21 | 31 | 41 | 51 |
| 2 | 02 | 12 | 22 | 32 | 42 | 52 |
| 3 | 03 | 13 | 23 | 33 | 43 | 53 |
| 4 | 04 | 14 | 24 | 34 | 44 | 54 |
| 5 | 05 | 15 | 25 | 35 | 45 | 55 |

○ C.

| | | | | |
|---|----|----|----|----|
| | 1 | 2 | 3 | 4 |
| 1 | 11 | 21 | 31 | 41 |
| 2 | 12 | 22 | 32 | 42 |
| 3 | 13 | 23 | 33 | 43 |
| 4 | 14 | 24 | 34 | 44 |

b) How many pairs of digits are there?

c) Which table shows the sample space if the first digit is either 1, 2, or 5?

☐ A.

| | 1 | 2 | 3 | 4 | 5 |
|---|----|----|----|----|----|
| 1 | 11 | 21 | 31 | 41 | 51 |
| 2 | 12 | 22 | 32 | 42 | 52 |
| 3 | 13 | 23 | 33 | 43 | 53 |
| 4 | 14 | 24 | 34 | 44 | 54 |
| 5 | 15 | 25 | 35 | 45 | 55 |

☐ B.

| | 1 | 2 | 5 |
|---|----|----|----|
| 1 | 11 | 21 | 51 |
| 2 | 12 | 22 | 52 |
| 3 | 13 | 23 | 53 |
| 4 | 14 | 24 | 54 |
| 5 | 15 | 25 | 55 |

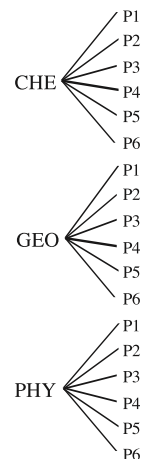
☐ C.

| | 1 | 2 | 5 |
|---|----|----|----|
| 1 | 11 | 21 | 51 |
| 2 | 12 | 22 | 52 |
| 5 | 15 | 25 | 55 |

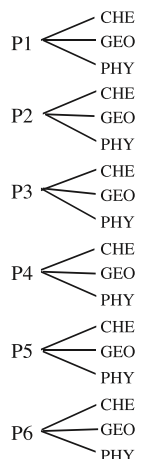
15. **Challenge** Heidi needs to take one of Chemistry (CHE), Geometry (GEO), or Physics (PHY) this year. She can take the class during any one of six periods (P1 through P6).

a) Which tree diagram displays the sample space if she chooses which period and then which class?

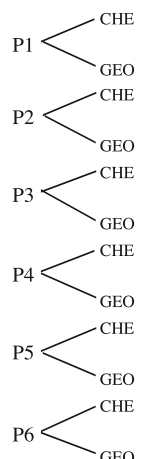
☐ A.



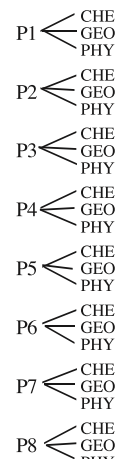
☐ B.



☐ C.



☐ D.



b) How many possible outcomes are there?

**Practice
17-3****Counting Outcomes**

1. A sales representative can take one of 2 different routes from City C to City F and any one of 3 different routes from City F to City H.
 - a) How many different routes can he take from City C to City H, going through City F?
 - b) Use F and a number to represent a route from City C to City F and use H and a number to represent a route from City F to City H. Which list below shows the possible routes?
 - ☐ A. {F1H1, F1H2, F1H3, F1H4, F1H5, F1H6, F2H1, F2H2, F2H3, F2H4, F2H5, F2H6}
 - ☐ B. {F1H1, F1H2, F1H3, F2H1, F2H2, F2H3, F3H1, F3H2, F3H3}
 - ☐ C. {F1H1, F1H2, F1H3, F2H1, F2H2, F2H3}
2. A restaurant offers 5 appetizers and 10 main courses. How many ways can a person order a two-course meal?
3. You have been asked to flip a coin for heads or tails and then select a golf ball from a bucket that contains 3 yellow golf balls and 4 white golf balls.
 - a) Use Y and a number to represent a yellow golf ball and W and a number to represent a white golf ball. Which list below shows the sample space?
 - ☐ A. {HW1, HW2, HW3, HY1, HY2, HY3, HY4, TW1, TW2, TW3, TY1, TY2, TY3, TY4}
 - ☐ B. {HY1, HY2, HY3, HW1, HW2, HW3, HW4, TY1, TY2, TY3, TW1, TW2, TW3, TW4}
 - ☐ C. {HY, HW, TY, TW}
 - b) How many ways can you expect to get heads and select a yellow golf ball?
4. At a restaurant you are going to order an appetizer and a main course. In how many different ways can you order a two-course meal that includes lasagna as the main course?

| Restaurant Menus | |
|-------------------|--------------|
| Appetizers | Main Course |
| Barbecue Wings | Hamburger |
| Chips and Salsa | Quesadilla |
| Mozzarella Sticks | Steak Tips |
| Mild Wings | Cheeseburger |
| Hot Wings | Lasagna |
| | Pizza |

5. Without repeating digits, form three-digit numbers using the digits 2, 3, 4, and 7.
- a) Which list shows the sample space?
- ☐ A. {222, 223, 224, 227, 232, 233, 234, 237, 242, 243, 244, 247, 272, 273, 274, 277, 322, 323, 324, 327, 332, 333, 334, 337, 342, 343, 344, 347, 372, 373, 374, 377, 422, 423, 424, 427, 432, 433, 434, 437, 442, 443, 444, 447, 472, 473, 474, 477, 722, 723, 724, 727, 732, 734, 737, 742, 743, 744, 747, 772, 773, 774, 777}
 - ☐ B. {234, 237, 247, 222, 347, 333, 444, 777}
 - ☐ C. {234, 237, 243, 247, 273, 274, 324, 327, 342, 347, 372, 374, 423, 427, 432, 437, 472, 473, 723, 724, 732, 734, 742, 743}
- b) How many of those three-digit numbers contain 4?
6. You are going to make a password using 7 of the letters followed by 3 of the digits of BLYJGUK 538916.
- a) Without repeating letters or digits, how many different passwords can you make?
- b) How many passwords can you make that begin with the letter G?
7. **Writing** You are going to pick a marble from a bag with 2 red marbles and 4 blue marbles. Then you are going to flip a coin for heads or tails.
- a) Use R and a number to represent a red marble and B and a number to represent a blue marble. Which list shows the sample space?
- ☐ A. {R1H, R2H, B1H, B2H, B3H, B4H, R1T, R2T, B1T, B2T, B3T, B4T}
 - ☐ B. {B1H, B2H, R1H, R2H, R3H, R4H, B1T, B2T, R1T, R2T, R3T, R4T}
 - ☐ C. {RH, RT, BH, BT}
- b) How many ways can you pick a blue marble and get heads?
- c) How would changing the number of blue marbles in the bag change the number of ways you can pick a blue marble and then get heads?
8. a) **Reasoning** Without repeating digits, how many four-digit numbers can you make using the digits 7, 3, 2, 5, 4, and 1?
- b) How many of those four-digit numbers begin with 3?
- c) Without repeating letters, how many seven-letter combinations can you make using the letters C, A, E, U, T, F and R?
- d) How many of those seven-letter combinations begin with E?
- e) Explain how the number of outcomes of a multi-step process is related to the number of outcomes in each step.

9. Error Analysis At a deli you can order a sandwich with one type of meat and one type of cheese from the menu. Tara incorrectly said you could order a sandwich that includes salami in 18 different ways.

| Sandwich Ingredients | |
|----------------------|---------------|
| Meat | Cheese |
| Salami | Monterey Jack |
| Ham | Provolone |
| Chicken | American |
| | Mozzarella |
| | Cheddar |
| | Swiss |

- a) In how many different ways can you order a sandwich that includes salami?
- b) What mistake might Tara have made?
- ☐ A. She subtracted the number of possible cheeses from the possible meats instead of adding.
 - ☐ B. She added the number of possible cheeses from the possible meats instead of subtracting.
 - ☐ C. She found the number of sandwiches that include provolone cheese instead of the number of sandwiches that include salami.
 - ☐ D. She found the number of all possible sandwiches instead of the number of sandwiches that include salami.

10. Passwords You want to create a three-letter password for a gadget using the letters S, R, L, and P without repeating any letter.

- a) Which list shows the sample space?
- ☐ A. {SRL, SRP, SLR, SLP, SPR, SPL, RSL, RSP, RLS, RLP, RPS, RPL, LSR, LSP, LRS, LRP, LPS, LPR, PSR, PSL, PRS, PRL, PLS, PLR}
 - ☐ B. {SSS, SSR, SSL, SSP, SRS, SRR, SRL, SRP, SLS, SLR, SLL, SLP, SPS, SPR, SPL, SPP, RSS, RSR, RSL, RSP, RRS, RRR, RRL, RRP, RLS, RLR, RLL, RLP, RPS, RPR, RPL, LRP, LLS, LLR, LLL, LLP, LPS, LPR, LPL, RPP, LSS, LSR, LSL, LSP, LRS, LRR, LRL, LPP, PSS, PSR, PSL, PSP, PRS, PRL, PRP, PLS, PLR, PLL, PLP, PPS, PPR, PPL, PPP}
 - ☐ C. {SRL, SRP, SLP, SSS, RLP, RRR, LLL, PPP}
- b) How many of those three-letter passwords contain P?

11. Mental Math The owner of a stereo store wants to advertise that she has many different sound systems in stock. The store carries 7 different CD players, 6 different receivers, and 10 different speakers. A sound system consists of one of each item. How many different sound systems can the store owner advertise?

12. Jessica's class schedule for next semester must consist of exactly one class from each of the four categories shown. All sections for the 3 most popular classes in Education are full. The rest of the courses are available. Determine the number of different sets of classes Jessica can take.

| Category | Number of Choices |
|-------------|-------------------|
| Economics | 3 |
| Mathematics | 3 |
| Education | 5 |
| Sociology | 5 |

13. a) Without repeating letters, create two-letter combinations using the letters Q, P, and N. Which list below shows the sample space of two-letter combinations?
- ☐ A. {QQ, QP, QN, PQ, PP, PN, NQ, NP, NN}
 - ☐ B. {QP, QN, PQ, PN, NQ, NP}
 - ☐ C. {QQ, QP, QN}
- b) How many of those two-letter combinations contain P?
- c) Without repeating digits, create three-digit numbers using the digits 3, 4, 5, and 7. Which list below shows the sample space of three-digit numbers?
- ☐ A. {345, 347, 354, 357, 374, 375, 435, 437, 453, 457, 473, 475, 534, 537, 543, 547, 573, 574, 734, 735, 743, 745, 753, 754}
 - ☐ B. {333, 334, 335, 337, 343, 344, 345, 347, 353, 354, 355, 357, 373, 374, 375, 377, 433, 434, 435, 437, 443, 444, 445, 447, 453, 454, 455, 457, 473, 474, 475, 547, 553, 554, 555, 557, 573, 574, 575, 477, 533, 534, 535, 537, 543, 544, 545, 577, 733, 734, 735, 737, 743, 745, 747, 753, 754, 755, 757, 773, 774, 775, 777}
 - ☐ C. {345, 347, 357, 333, 457, 444, 555, 777}
- d) How many of those three-digit numbers begin with 7?
14. **Challenge** In a bag of marbles there are 4 blue marbles, 3 red marbles, and 4 green marbles. You pick three marbles out of a bag one at a time without replacement. How many ways can you pick two blue marbles first and one red marble last?
15. a) **Challenge** Without repeating any characters, how many eight-character combinations can you make using the letters and digits LWRNPXZ 285137?
- b) How many of those combinations begin with N?